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THE KINZUA VIADUCT OF THE ERIE RAILROAD COMPANY.

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Presented January 2d, 1901.

WITH DISCUSSION.

The Kinzua Viaduct is on the Bradford Division of the Erie Railroad, and is situated in McKean County, Pa.

The old viaduct was in service for about nineteen years, and has now been replaced by a new steel structure, built under the charge of C. W. Buchholz, M. Am. Soc. C. E., Chief Engineer of the Erie Railroad Company, Mr. M. Strong being the Bridge Engineer. The contractors for the steel work were the Elmira Bridge Company, Limited, Elmira, N. Y.; and the contractors for the erection were Grattan and Jennings, Buffalo, N. Y. The drawings were prepared under the care of E. E. Buchanan, M. Am. Soc. C. E., Secretary of the Elmira Bridge Company. The writer, in his capacity as Engineer of that company, designed the viaduct and also suggested the particular nature of its towers, which, in their construction, form a radical departure from previous practice in viaduct designing. The character of the tower bents is not new, as such forms have often

been used by structural as well as mechanical engineers; but their application to a railway structure and on a large scale is certainly a novelty.

The design of the viaduct is quite simple, so that it will suffice to give a brief description, but before proceeding to do so, it may be well to state the reasons which led the writer to make his suggestions.

The original problem consisted of the design of towers with stiff members throughout, together with secondary struts and diagonals for the taller towers, lying in a longitudinal vertical plane, coincident with the axis of the structure, and forming a support for the very long members of the wind bracing. A design for the tallest tower, in conjunction with an estimate of its weight, was prepared by the writer, but the weight proved to be excessively great, even after many changes in the design had been introduced. The next step was the substitution of lateral rods for all the longitudinal and transverse diagonals, but this design, too, although calling, of course, for less weight, was soon abandoned. In the early stages of the design the writer entertained no doubt that a tower of nearly 300 ft. in height, and based on the lines described, would never give the desired results, but that, on the contrary, they could be effected by a radical change.

Such tall towers have far too many members to conform with all those advantages which are sought to be realized in a structure. The secondary members, running to a very great weight, and solely designed as a crutch for the protection of the wind bracing, did not appear to the writer as simple means to accomplish a certain purpose. On the contrary, he thought that this purpose could be attained more simply and more directly by building the wind bracing strong enough to take care of itself without the support of a secondary bracing, and likewise, he thought that the tower legs, being the real supporters of the railway, should stand out boldly, instead of looking rather slender and more or less overshadowed by a great mass of bracings. But columns of large size meant a considerable increase in the height of the stories, fewer members to build, a quicker erection and a great reduction in field riveting.

For these reasons the writer suggested two different forms of tower bents, one, as actually built, without transverse diagonals, and the

FIG. 1.—THE OLD KINZUA VIADUCT AND TRAVELER.

FIG. 2.—NEW KINZUA VIADUCT IN PROCESS OF ERECTION.

other as shown in Fig. 1, in both of which a secondary bracing is dispensed with.

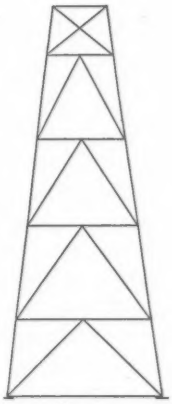


FIG. 1.

Estimates of the weight of each frame were made, and, although the rejected form of bent has merits well worth full consideration, it may be dismissed at once, as its discussion is outside the province of this paper. The following is a brief description of the work:

The great reduction of the bending moments, due to the wind forces, and a consequent reduction in the weight of the towers, is due to the fact that each bent of the accepted form is practically a triangle, with the principal wind force acting at the top, the sides being rigidly united at the apex by a header, 8 ft. 10 ins. deep, made up of two solid plates as webs, the chords of which are built of angles. These facts, taken in connection with the general appearance of the frame, have favored the adoption of this form of bent.

The viaduct, resting on the old piers, is 2 053 ft. long, and carries the railway on twenty towers varying in height from 30 ft. to about 285 ft., from the top of the masonry to the base of the rail, and having a batter of 1 to 6. Fig. 2 is an elevation of the highest bent. As mentioned before, no secondary bracing has been used, but the tallest towers are provided with horizontal adjustable bracings to line up the tower legs during erection. The dimensions, sizes and details of all the towers are the same from their tops downward, or in other words, the dimensions, sizes, etc., for any tower are obtained from the tallest tower by shortening the same from the bottom upward.

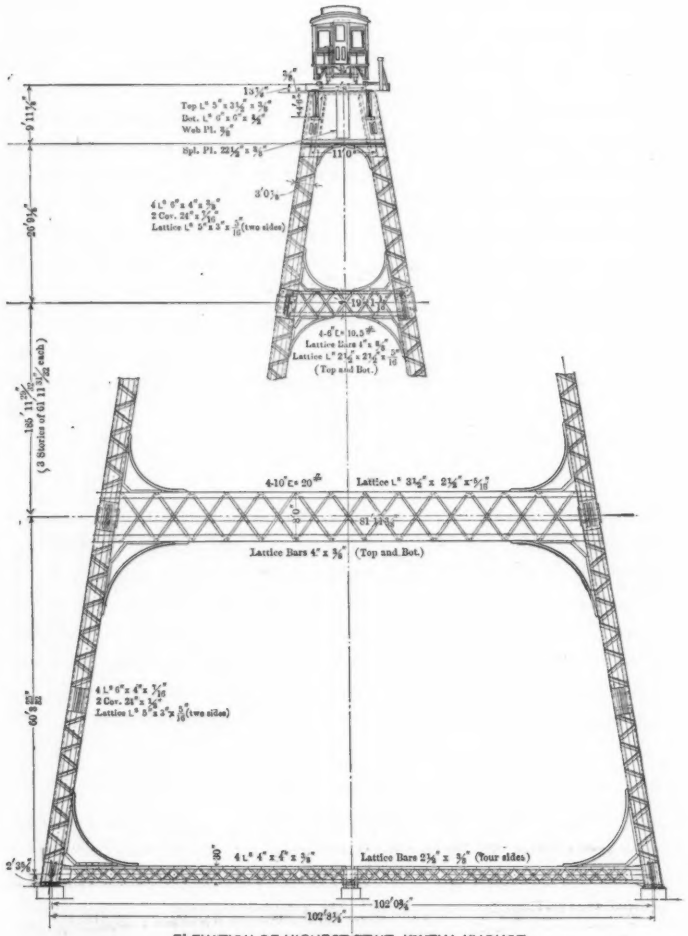
The height of the stories, taken transversely, is 62 ft., which means an increase in height of about 100% over that which is common in viaducts, while the height on the longitudinal faces is 31 ft. The columns have no bents at the top, and are composed of two plates, 2 ft. wide, and four 6 x 4-in. angles. They measure 3 ft. 1 in. transversely, and are laced by 5 x 3-in. angle bars. Through each end of the lacing angles, in the two bottom stories of the tallest towers, pass four $\frac{7}{8}$-in. rivets, while only three $\frac{7}{8}$-in. rivets have been used for the three top stories. The bottom struts are deep angle struts, latticed on all four

sides, and the higher bents are bolted down at their middle to a pier. The depths of the intermediate transverse struts are 4, 6, 7 and 8 ft., respectively. They have been designed as box lattice girders with stiff web members and brackets at each end, portal fashion. Each chord of these struts is composed of two channels, and their ends, as well as those of the headers, are shaped fork-like so that the columns can pass between the channels of the struts and the web plates of the headers, by which means substantial and very rigid connections are obtained. The single brackets, the legs of which have an average length of 12 ft., have solid webs, and where they connect to the columns, the plates of the latter are stiffened by angles. The longitudinal diagonals form a double system, and are built of two latticed channels connected with plates at their intersection points, and where these diagonals connect with the columns the latter are provided with two diaphragms, about 2 ft. apart. Each tower has two longitudinal bottom struts, but no other longitudinal struts, which make these systems complete without the tower stringers.

The tower legs are bolted to the masonry by two 1¼-in. bolts, which had been used in the old viaduct, and which vary in length from 6 ft. to about 10 ft. Provision to meet the effects of changes in temperature is made by pockets, riveted to the headers, in which the ends of the intermediate spans can slide. There are in all twelve freely moving girder ends in the viaduct, which are obtained in the following manner: Each end span is free to move, as also the intermediate span between the two smallest towers at each end of the viaduct, and five pairs of towers are united by having their intermediate spans riveted tightly between them, and, likewise, two groups of three towers each are riveted together. In this way the stresses arising from the braking of trains are reduced. The columns of the three smallest towers at each end of the structure are furnished with sliding plates, while the others are provided with nests of friction rollers.

The fixed ends of the tower legs are placed on the north side of the viaduct, as the wind blows chiefly from that direction.

The spans, Fig. 3, are plate-girder deck-spans, 9 ft. on centers, with the tower spans riveted between the headers. The depth of the tower spans is 4 ft. 6 ins., and the length 38 ft. 6 ins. on centers, and for the intermediate spans the depth is 6 ft. 6 ins. and the length 61 ft. on centers. The chords of the stringers are made up of two 6 x 6-in.


Top L.s 5" x 3½" x 5/16"
Bot. L.s 6" x 6" x 5/8"
Web Pl. 5/8"
Spl. Pl. 22 5/16" x 5/8"
4 L.s 6" x 4" x 5/8"
2 Cov. 24" x 1/2"
Lattice L.s 5" x 3" x 5/16" (two sides)
4-6"C= 10.9#
Lattice Bars 4" x 5/8"
Lattice L.s 2½" x 2½" x 5/16"
(Top and Bot.)
4-10"C= 20#
Lattice L.s 3½" x 2½" x 5/16"
Lattice Bars 4" x 5/8" (Top and Bot.)
4 L.s 6" x 4" x 1/2"
2 Cov. 24" x 1/2"
Lattice L.s 5" x 3" x 5/16" (two sides)
4 L.s 4" x 4" x 5/8"
Lattice Bars 2½" x 5/8" (four sides)
(3 Stories of 61' 11 25/32" each)
102'03¼"
102'03¼"

ELEVATION OF HIGHEST BENT KINZUA VIADUCT,

FIG. 2.

angles and one coverplate 14 ins. wide, which runs the full length, both top and bottom, of all stringers. The spans are provided with stiff top and vertical bracings, but bottom bracings are omitted. Other dimensions and sizes, together with the principal details, are shown by the illustrations, so that it will not be necessary to refer to them.

The maximum shearing force in the webs of the plate girders is 4 000 lbs. per square inch cross-section; the maximum flange stress in the intermediate girders equals 12 000 lbs. per square inch, and that of the tower girders 11 600 lbs. The maximum stress in the towers from the supposed combinations of loading does not exceed 14 000 lbs. per square inch, while the elastic limit of the steel used is between 33 000 and 35 000 lbs.

Under the assumed loadings the anchor bolts are never called into action. The shipping weight of the deck spans complete is 638 tons and that of the towers 2 715 tons, making the total weight of the viaduct 3 353 tons. The structure was put into service about the middle of September, 1900.

Erection.

The work of erecting the viaduct has been carried on simultaneously from each end by two travelers. These carried the hoisting machinery and spanned a clear space of 160 ft., having an old tower in their middle. In this position the old tower has been taken apart, its material hoisted up and loaded on cars to be run out to the shore end of the structure, whereupon a new tower could be assembled. The work of removing the old viaduct and erecting the new one consumed four months, with a force of about 120 men at 10 hours a day.

Specifications.

The viaduct was built under the regular Erie Railroad specifications for bridges, with certain modifications as to allowable stresses, as follows: Unit stresses for live and dead loads were increased 25%, and in the flanges of the girders the whole cross-section was considered as available, but the webs of girders are spliced for flange stress, although not considered in flange section.

Where combinations of wind stresses—bending and direct—with dead-load stresses, or combinations of all the above with live-load

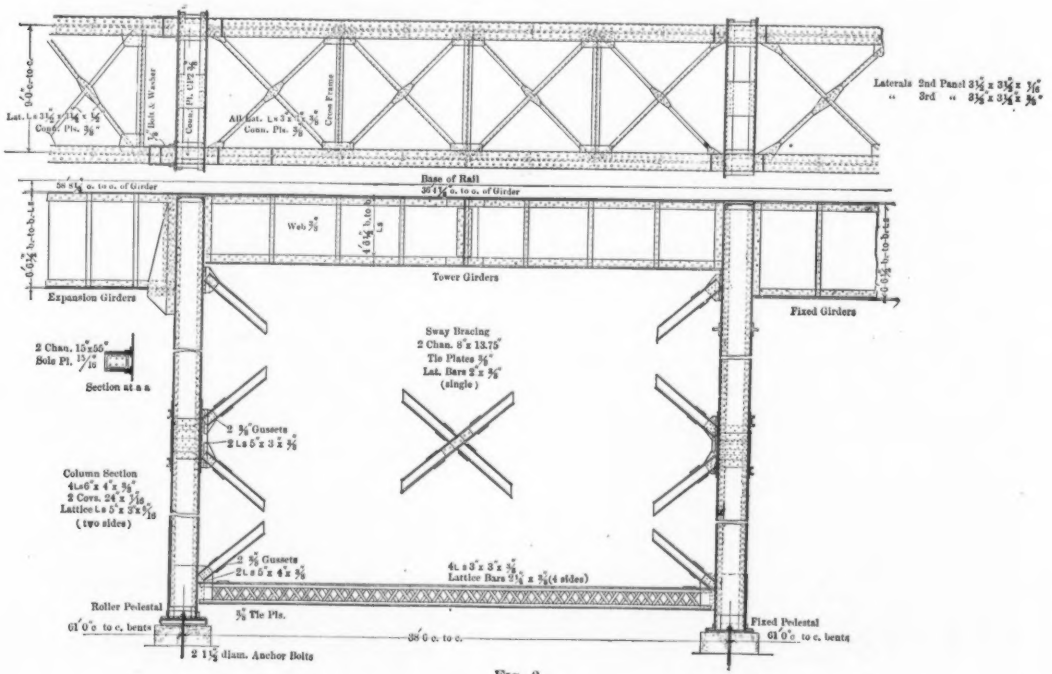

Laterals 2nd Panel 3½ x 3½ x ⅜"
" 3rd " 3½ x 3½ x ⅜"
Base of Rail
Web ⅝"
Tower Girders
Expansion Girders
Fixed Girders
Sway Bracing
2 Chan. 8" x 13.75"
Tie Plates ⅝"
Lat. Bars 2" x ⅜"
(single)
2 ⅝" Gussets
2 L s 5" x 3½ x ⅝"
Column Section
4 L s 6" x 4" x ⅝"
2 Covs. 24" x ⅜"
Lattice L s 5" x 3½ x ⅜"
(two sides)
2 ⅝" Gussets
2 L s 5" x 4" x ⅝"
4 L s 3" x 3" x ⅜"
Lattice Bars 9½" x 2¾" (4 sides)
Roller Pedestal
61' 0" c. to c. bents
⅝" Tie Pls.
38' 0" c. to c.
Fixed Pedestal
61' 0" c. to c. bents
2 1¾" diam. Anchor Bolts
Section at a a

FIG. 3.

stresses, or combinations of dead-load stresses with live-load stresses, and traction stresses from the live load exist, the stresses were not allowed to exceed the foregoing by more than 50 per cent.

The wind pressure for the loaded structure was considered to be 100 lbs. per vertical foot of bent plus 30 lbs. per square foot on floor and train. That for the light structure was considered to be 160 lbs. per vertical foot of bent plus 50 lbs. per square foot on floor.

Method of Calculating.

There is no group of stresses which calls for any special mention, except that which is due to the wind forces, as these latter are resisted by bending as well as by direct stresses, and for this reason this group of stresses alone will be investigated.

As far as known, the stresses in frames of the kind under consideration have never before been analytically discussed, and it may be noted that, in order to effect a solution, the problem can be approached from different points of view. The writer preferred a modern analytical method, proceeding from the principle of least work, but, before explaining the method of application of this principle, some general remarks in regard to these tower bents will be given.

In computing the stresses in a bent it is customary to proceed from the supposition that it acts like a cantilever, fixed at the bottom and otherwise free to move; but, while the influence of a force that acts on a bent with articulated joints is confined to a field extending from the point of application of the force to the bottom of the bent, a force that acts on a bent with fixed joints, no matter where its point of application, exercises an influence over the entire frame. A bent of only one story, fixed at the bottom and with fixed joints, will be considered first.

Such a frame partakes of the nature of a hingeless arch, and if quite a general case of loading be assumed, then this frame, like the hingeless arch, is threefold statically indeterminate; that is to say, the values of three unknown quantities must first be found before the stresses can be computed. For this supposed condition of loading none of the points of contraflexure is at the center of the different members, and those of the columns are besides in different horizontal planes. But this state of things is changed, if we have to deal with special loadings, as, for instance, the wind forces, which are supposed to act as concentrated loads at the panel points and in horizontal

FIG. 1.—NEW KINZUA VIADUCT IN PROCESS OF ERECTION.

FIG. 2.—THE NEW KINZUA VIADUCT.

planes. In this case the frame becomes singly statically indeterminate. The bending moments of the columns are alike, and the points of contraflexure are on the same level, but not in the middle of these members, which points to the fact that the direct stress in the header equals half of the exterior force, and with the point of contraflexure at the center of the header.

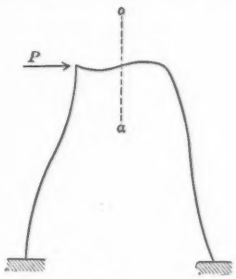


FIG. 4.

In order to illustrate the foregoing statements we pass a cut $\alpha\ \alpha$ through the center of the bent, as indicated in Fig. 4, and apply at this point the unknown vertical force X, Fig. 5, acting upward in the left fragment and downward in the right fragment of the frame. We also apply $\frac{P}{2}$ to each fragment, but a moment need not be applied at the center, because no moment can pass that point. The problem is now to determine the value of the unknown X. In like manner, in a two-story bent the direct stress in the header and in the intermediate strut is equal to half of their exterior concentrations, and their points of contraflexure are in the middle, while the bending moments of corresponding columns are alike and their points of contraflexure are not in the middle. The number of unknowns in a two-story bent is therefore 2, and in a bent of n stories the number is n.

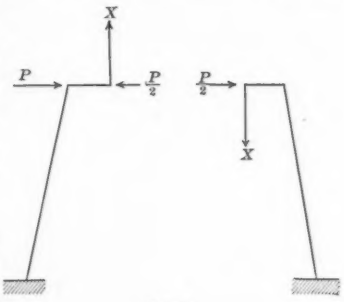


FIG. 5.

The problem can also be stated to consist of finding the locations of the points of contraflexure, because, if these locations were known, it would only be necessary to resolve the resultant of the exterior forces into the reactions, which must pass through the points of contraflexure, and, with the reactions known, the direct stress in any member and the bending moment in any point of the frame is easily computed.

Aside from the margin of safety, provided for by the specifications,

judging solely from his previous experience with frames of a similar nature, the writer felt satisfied that this graphical method would answer all practical purposes. In fact, he planned it for practical use, and only his desire to know how much the results thus obtained would vary from the analytical results prompted him to develop an analytical treatment.

The writer made the calculations for a wind pressure of 30 lbs. per square foot on girders and train and 100 lbs. per vertical foot of bent, and for five different bents, varying in height by a regular story of 62 ft., excepting the top story, which has a height of about 31 ft., and the results thus obtained answer perfectly well for the bents in the entire structure. Considering the magnitude of the stresses thus computed, an approximate determination of the effects of a wind force of 50 lbs. per square foot on the girders and 160 lbs. per vertical foot of bent appeared to be justifiable. After these few remarks the application of the principle of least work to the bents will now be shown, but, in doing so, the reader's familiarity with this principle must be assumed, as a demonstration of its truth is beyond the scope of this paper.

The bents are considered as being under no initial stress. Neglect the small alterations in the lengths of the members and consider bending alone. The principle of least work requires that the work of deformation shall be a minimum, which is expressed by

$$W = \int \frac{M^2}{2\,E\,I}\,dx = \text{Min.}$$

In this term W = work of deformation, M = moment of the exterior forces at any point, E = modulus of elasticity, supposed to be constant throughout the frame, and I = constant moment of inertia of a member under consideration. Now, if the work of deformation be a minimum, its partial derivatives, taken successively with respect to the unknowns $X_1, X_2 \ldots\ldots X_n$, treated as independent of each other, must be put each equal to zero. The equations of condition are then as follows:

$$\left.\begin{aligned}
\frac{\delta W}{\delta X_1} &= \frac{\delta\left[\int \frac{M^2}{2\,E\,I}\,dx\right]}{\delta X_1} = \int \frac{M}{E\,I}\,\frac{\delta M}{\delta X_1}\,dx = 0 \\
\frac{\delta W}{\delta X_2} &= \frac{\delta\left[\int \frac{M^2}{2\,E\,I}\,dx\right]}{\delta X_2} = \int \frac{M}{E\,I}\,\frac{\delta M}{\delta X_2}\,dx = 0 \\
\frac{\delta W}{\delta X_n} &= \frac{\delta\left[\int \frac{M^2}{2\,E\,I}\,dx\right]}{\delta X_n} = \int \frac{M}{E\,I}\,\frac{\delta M}{\delta X_n}\,dx = 0
\end{aligned}\right\} \ldots\ldots A$$

FIG. 1.—KINZUA VIADUCT.—CENTRAL VIEW THROUGH TOP STORY.

FIG. 2.—KINZUA VIADUCT.—CENTRAL VIEW THROUGH AN INTERMEDIATE STORY.

The equations "A" are directly applicable to the bents, but it must be noted that the integrations have to be extended over the whole frame, which is equivalent to saying that the integrals, after they have been computed with respect to the individual members of the frame, must all be added together. The reader is now referred to the stress sheet, Fig. 9, and to the table on pages 34 and 35, which the writer prepared for the sake of convenience. The stress sheets and the table contain all the data necessary for the computations, and by the use of the table the different sets of the equations of condition can readily be written down at once; but it should be remarked that each computed integral, as given in the table, must be divided by its corresponding moment of inertia and also by E, the modulus of elasticity. E, being a constant, drops out of the equations, if these are all multiplied by E.

Returning to the one-story bent and referring to the stress sheets, the method of computation is now as follows:

Header:

$$M = -X_5\, r; \quad \frac{\delta M}{\delta X_5} = -r.$$

$$\int \frac{M\,\delta M}{\delta X_5}\, d x = \int_{o}^{h\,=\,4.765} X_5\, r^2\, d r = 36.06\, X_5.$$

Column:

$$M = P_5 z - X_5 (h + n z); \quad \frac{\delta M}{\delta X_5} = -(h + n z)$$

$$\int \frac{M\,\delta M}{\delta X_5}\, d x = -\int_{a\,=\,31.16} [P_5 z - X_5 (h + n z)]\, (h + n z)\, d z =$$
$$-26760.27 + 1758.54\, X_5$$

The equation of condition becomes now:

$$\frac{36.06\, X_5}{107500} - \frac{26760.27 - 1758.54\, X_5}{10750} = 0$$

and its solution gives: $X_5 = 15.2$ tons. Likewise, the two equations of condition for a two-story bent are found from the table, viz.:

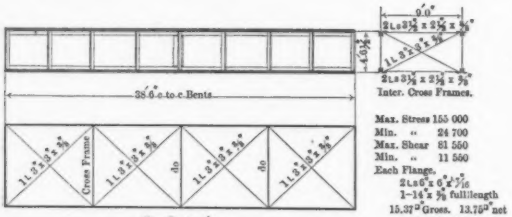
$$\frac{-26760.27 + 1758.54\, X_5 - 451582.21 + 14735.11\, [X_5 + X_4]}{10750} + \frac{36.06\, X_5}{107500} = 0.$$

$$\frac{-451582.21 + 14735.11\, [X_5 + X_4]}{10750} + \frac{329.35\, X_4}{5511} = 0$$

The values of the unknowns are: $X_5 = 19.27$ tons and $X_4 = 10.9$ tons.

After the unknowns have been determined, it is then easy to calculate the bending moments at every point of the frame.

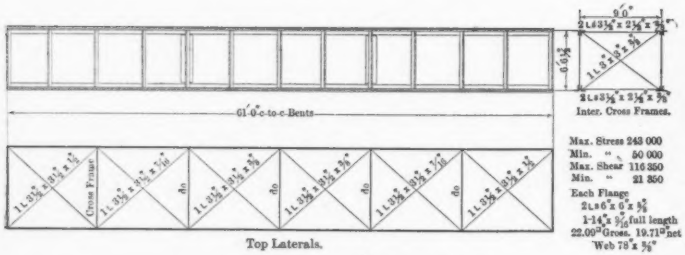
The direct stresses in the columns $= \mp \sin. \alpha \Sigma P \mp \cos. \alpha \Sigma X$, and they are tension on the windward and compression on the leeward side of the bent, α being the angle formed by the axis of the tower leg and the vertical. The shear on the columns is principally and effectively resisted by the lacing angles.


9′0″
2 L s 3½ x 2½ x 5/16
60½″
1 L 3″ x 3″ x 5/16
2 L s 3½ x 2½ x 5/16
Inter. Cross Frames.
38′6″ c-to-c Bents
Cross Frame
do
do
Max. Stress 155 000
Min. " 24 700
Max. Shear 81 550
Min. " 11 550
Each Flange.
2 L s 6″ x 6″ x 7/16
1–14″ x 5/16 full length
15.37 in² Gross. 13.75 in² net
Web 54″ x 5/16″.

Top Laterals.

Assumed Dead Load=1200 lbs. per lin. foot of span.

FIG. 6.



Top Laterals.

Assumed Dead Load=1400 lbs. per lin. foot of span.

FIG. 7.

Bent of 1 story.....$X_5 = 15.2$.
" " 2 stories.....$X_5 = 19.27$; $X_4 = 10.9$.
" " 3 " .....$X_5 = 19.41$; $X_4 = 11.64$; $X_3 = 7.77$.
" " 4 " .....$X_5 = 19.43$; $X_4 = 11.67$; $X_3 = 8.10$; $X_2 = 5.86$.
" " 5 " .....$X_5 = 19.44$; $X_4 = 11.68$; $X_3 = 8.14$; $X_2 = 6.2$; $X_1 = 4.96$.

In the manner just outlined the computations for the five different bents have been made, and, although it was somewhat tedious, it in no wise proved to be a hardship. The structure of the equations of condition is such that those for a four and five-story bent have been effected by ordinary means, instead of having recourse to the theory of determinants.

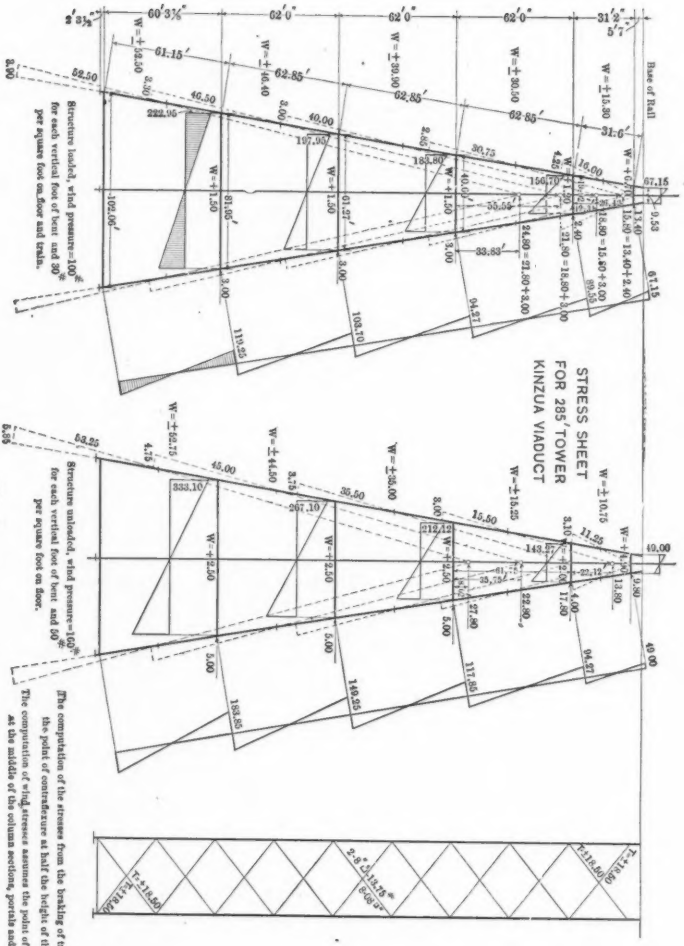
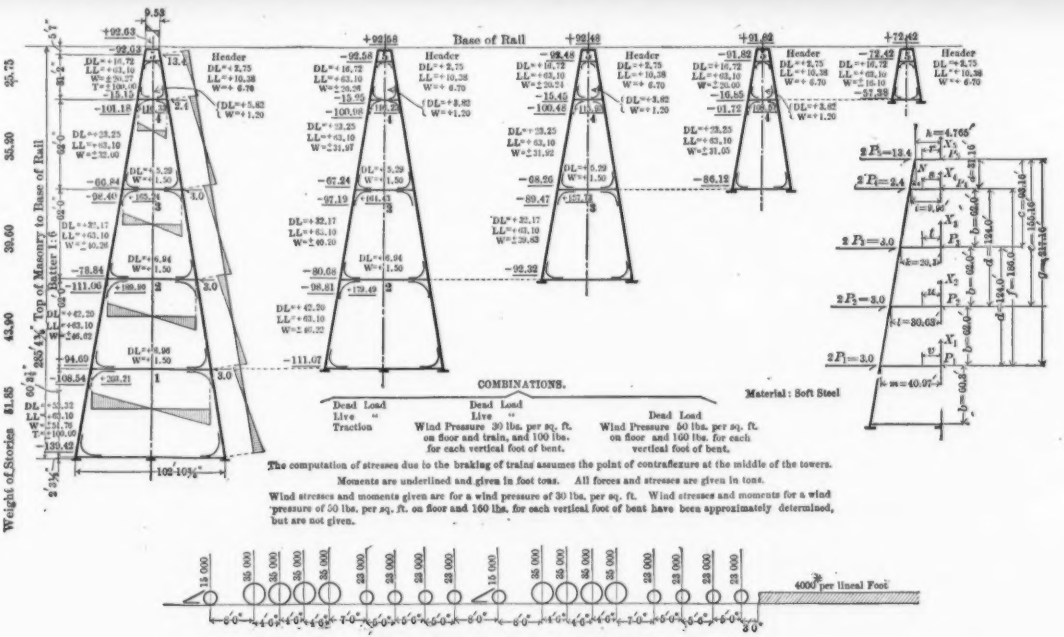

STRESS SHEET
FOR 285' TOWER
KINZUA VIADUCT
Base of Rail
Structure loaded, wind pressure=100# per square foot on floor and train.
Structure unloaded, wind pressure=100# per square foot on floor.
The computation of the stresses from the breaking of trains assumes the point of contraflexure at half the height of the towers.
The computation of wind stresses assumes the point of contraflexure at the middle of the column sections, portals and headers.

Fig. 8.

Sizes.	1st-story column.	Header.	Strut "4." Twin latt. girder.	Strut "3." Twin latt. girder.	Strut "2." Twin latt. girder.	Strut "1." Twin latt. girder.
	4 ⌊'s 6 x 4 x $\frac{7}{16}$ in. 2 Pl. 24 x $\frac{1}{2}$ in. Latt. L's 5 x 3 x $\frac{5}{16}$ in.	2 L's 5 x $3\frac{1}{2}$ x $\frac{3}{8}$ in. 2 L's 6 x 6 x $\frac{1}{2}$ in. 2 Pl. $105\frac{1}{2}$ x $\frac{3}{8}$ in.	4 6" U's 10.5 lb. Latt. L's $2\frac{1}{2}$ x $2\frac{1}{2}$ x $\frac{5}{16}$ in. Latt. bars 4 x $\frac{3}{8}$ in.	4 8" U's 13.75 lb. Latt. L's $2\frac{1}{2}$ x $2\frac{1}{2}$ x $\frac{5}{16}$ in. Latt. bars 4 x $\frac{3}{8}$ in.	4 10" U's 20.0 lb. Latt. L's $3\frac{1}{2}$ x $2\frac{1}{2}$ x $\frac{5}{16}$ in. Latt. bars 4 x $\frac{3}{8}$ in.	4 10" U's 20.0 lb. Latt. L's $3\frac{1}{2}$ x $2\frac{1}{2}$ x $\frac{5}{16}$ in. Latt bars 4 x $\frac{3}{8}$ in.
Width and depth out to out	2' 0½" x 3' 1"	2' 1¼" x 8' 10"	2' 4⅝" x 4' 0"	2' 5½" x 6' 0"	2' 6" x 7' 0"	2' 6" x 8' 0"
Moment of inertia	12 356	107 500	5 511	16 692	32 514	43 803
Unknowns		X_5	X_4	X_3	X_2	X_1
Moment M of exterior forces	$P_5 g + P_4 f + P_3 d + P_2 b (P_5 + P_4 + P_3 + P_2 + P_1) z - (X_5 + X_4 + X_3 + X_2 + X_1)(m + n z)$	$- X_5 r$	$- X_4 s$	$- X_3 t$	$- X_2 u$	$- X_1 v$
$\frac{\delta M}{\delta X_5}$	$- (m + n z)$	$- r$				
$\frac{\delta M}{\delta X_4}$	$- (m + n z)$		$- s$			
$\frac{\delta M}{\delta X_3}$	$- (m + n z)$			$- t$		
$\frac{\delta M}{\delta X_2}$	$- (m + n z)$				$- u$	
$\frac{\delta M}{\delta X_1}$	$- (m + n z)$					$- v$
$\int M \frac{\delta M}{\delta X_5} dx$	$- 6502866.84 + 128073.93 (X_5 + X_4 + X_3 + X_2 + X_1)$	$+ 36.06 X_5$				
$\int M \frac{\delta M}{\delta X_4} dx$	$- 6502866.84 + 128073.93 (X_5 + X_4 + X_3 + X_2 + X_1)$		$+ 329.35 X_4$			
$\int M \frac{\delta M}{\delta X_3} dx$	$- 6502866.84 + 128073.93 (X_5 + X_4 + X_3 + X_2 + X_1)$			$+ 2788.47 X_3$		
$\int M \frac{\delta M}{\delta X_2} dx$	$- 6502866.84 + 128073.93 (X_5 + X_4 + X_3 + X_2 + X_1)$				$+ 9579.00 X_2$	
$\int M \frac{\delta M}{\delta X_1} dx$	$- 6502866.84 + 128073.93 (X_5 + X_4 + X_3 + X_2 + X_1)$					$+ 22923.27 X_1$

Sizes.	5th-story column.	4th-story column.	3d-story column.	2d-story column.
	4 ⌊'s 6 x 4 x $\frac{3}{8}$ in. 2 Pl. 24 x $\frac{7}{16}$ in. Latt. ⌊'s 5 x 3 x $\frac{5}{16}$ in.	4 ⌊'s 6 x 4 x $\frac{3}{8}$ in. 2 Pl. 24 x $\frac{7}{16}$ in. Latt. ⌊'s 5 x 3 x $\frac{5}{16}$ in.	4 ⌊'s 6 x 4 x $\frac{3}{8}$ in. 2 Pl. 24 x $\frac{7}{16}$ in. Latt. ⌊'s 5 x 3 x $\frac{5}{16}$ in.	4 ⌊'s 6 x 4 x $\frac{7}{16}$ in. 2 Pl. 24 x $\frac{1}{2}$ in. Latt. ⌊'s 5 x 3 x $\frac{5}{16}$ in.
Width and depth out to out	2′ 0$\frac{1}{2}$″ x 3′ 0$\frac{7}{8}$″	2′ 0$\frac{1}{2}$″ x 3′ 0$\frac{7}{8}$″	2′ 0$\frac{1}{2}$″ x 3′ 0$\frac{7}{8}$″	2′ 0$\frac{1}{2}$″ x 3′ 1″
Moment of inertia	10 750	10 750	10 750	12 356
Unknowns				
Moment M of exterior forces	$P_5 z - X_5 (h + n z)$	$P_5 a + (P_5 + P_4) z - (X_5 + X_4)(i + n z)$	$P_5 c + P_4 b + (P_5 + P_4 + P_3) z - (X_5 + X_4 + X_3)(k + n z)$	$P_5 e + P_4 d + P_3 b + (P_5 + P_4 + P_3 + P_2) z - (X_5 + X_4 + X_3 + X_2)(l + n z)$
$\frac{\delta M}{\delta X_5}$	$-(h + n z)$	$-(i + n z)$	$-(k + n z)$	$-(l + n z)$
$\frac{\delta M}{\delta X_4}$		$-(i + n z)$	$-(k + n z)$	$-(l + n z)$
$\frac{\delta M}{\delta X_3}$			$-(k + n z)$	$-(l + n z)$
$\frac{\delta M}{\delta X_2}$				$-(l + n z)$
$\frac{\delta M}{\delta X_1}$				
$\int M \frac{\delta M}{\delta X_5} d x$	$-23760.27 + 1758.54 X_5$	$-451582.21 + 14735.11 (X_5 + X_4)$	$-1594211.87 + 40761\ 72 (X_5 + X_4 + X_3)$	$-3629878.5 + 79998.14 (X_5 + X_4 + X_3 + X_2)$
$\int M \frac{\delta M}{\delta X_4} d x$		$-451582.21 + 14735.11 (X_5 + X_4)$	$-1594211.87 + 40761.72 (X_5 + X_4 + X_3)$	$-3629878.5 + 79998.14 (X_5 + X_4 + X_3 + X_2)$
$\int M \frac{\delta M}{\delta X_3} d x$			$-1594211.87 + 40761.72 (X_5 + X_4 + X_3)$	$-3629878.5 + 79998.14 (X_5 + X_4 + X_3 + X_2)$
$\int M \frac{\delta M}{\delta X_2} d x$				$-3629878.5 + 79998.14 (X_5 + X_4 + X_3 + X_2)$
$\int M \frac{\delta M}{\delta X_1} d x$				


Base of Rail
Weight of Stories
Top of Masonry to Base of Rail
Header
COMBINATIONS.
Dead Load
Live "
Traction
Dead Load
Live "
Wind Pressure 30 lbs. per sq. ft. on floor and train, and 100 lbs. for each vertical foot of bent.
Dead Load
Wind Pressure 50 lbs. per sq. ft. on floor and 160 lbs. for each vertical foot of bent.
Material: Soft Steel
4000 per lineal Foot

The computation of stresses due to the braking of trains assumes the point of contraflexure at the middle of the towers.

Moments are underlined and given in foot tons. All forces and stresses are given in tons.

Wind stresses and moments given are for a wind pressure of 30 lbs. per sq. ft. Wind stresses and moments for a wind pressure of 50 lbs. per sq. ft. on floor and 160 lbs. for each vertical foot of bent have been approximately determined, but are not given.

STRESS SHEET; KINZUA VIADUCT

FIG. 9.

The stress sheets show also the direct stresses and bending moments of the highest bent from the wind forces found by graphic statics as explained before, under the assumption that the points of contraflexure, through which the reactions must pass, are in the middle of each member of the bent.

The forces are all given in tons and the moments in foot-tons. The algebraic sum of the moments around a panel point must equal zero. Roughly speaking, the moments of the struts are about twice as great as those of the columns.

The author made a special investigation, which brought out the fact that the moments of inertia of a group of members may be changed as much as 100% without materially altering the final results.

DISCUSSION.

Mr. Morison. GEORGE S. MORISON, Past-President, Am. Soc. C. E. (by letter).—The structure by which the old Kinzua Viaduct has been replaced, while containing much of the modern practice of detail connections and riveting, differs from other viaducts in two radical features. The first of these lies in the fact that the tops of the two battered posts of each bent are brought so near together, and are so rigidly connected, that the bent instead of being a quadrilateral is treated as an isosceles triangle. The second peculiarity is that all transverse diagonal bracing is omitted, the posts being stiffened only by their own unusually large transverse dimensions, assisted by the cross-struts and the rigidity of the connections between those cross-struts and the posts.

The first of these features is to be commended. The ideal trestle bent would be an isosceles triangle carrying a superstructure so narrow that the weight is all taken at a single point on top. While this cannot be done, the design of the new Kinzua Viaduct approaches sufficiently near to it for practical purposes. With bents of such moderate height that intermediate stiffening of the posts becomes unnecessary, this form of construction seems all that can be desired. There are, however, two very important limitations: The first of these is in height, new complications being introduced whenever transverse stiffeners are required; and the second is that it is only applicable to single-track structures, the eccentric loading of a double-track bridge producing irregular conditions at the top of the structure.

The second peculiar feature is of a more radical type. In the usual design each bent becomes a cantilever truss, standing vertically instead of horizontally; anchored, by weight or otherwise, to the masonry foundations, resisting a horizontal instead of a vertical load, and with its stiffness determined by the unit strains in the two chords and the distance between those chords. In the Kinzua design the cantilever form is retained, but the cantilever, instead of resisting horizontal forces for its whole length, resists them only at the top. The struts at the intermediate points are simply stiffeners, their only function being to control the curves of flexure of the posts. This flexure is naturally due to two causes: The first is wind; the second is the usual perversity of all compression members in buckling under loads, which perversity increases with the length of the member. Were the transverse struts connected to the posts by pins, their effect as stiffeners would be comparatively small, as the transverse deflections of the two posts would be represented by approximately parallel curves, the only effect of the stiffeners being to maintain the parallelism of these curves. Such struts would undoubtedly increase the

stiffness of the posts, considered simply as compression members, but would add little to their ability to resist wind. In this case, however, the struts are rigidly attached to the posts, so that the angle made by the strut and the post must always remain constant. This introduces new conditions. If the struts were absolutely stiff (an impossible supposition) the direction of the posts at their points of intersection would always remain unchanged, but the posts of the lower part of each panel would bend from the wind in a curve the center of which is on the leeward side, recovering their direction in the upper part of each panel by a curve the center of which is on the windward side. As a moderate amount of flexure takes place also in the transverse struts, the actual bending in the lower part of the panel exceeds the recovery in the upper part of the panel. The result, on either supposition, is a series of cumulative deflections. The top of the lowest panel is deflected by an amount determined by the wind pressure and the stiffness of the posts. The second panel has its own similar deflection, but, as the base of the second panel is the top of the lower panel, the deflection of the top of the second panel is the sum of the two deflections; and so on to the top of the tower. The principle on which this structure appears to have been designed is that these various bendings of the posts shall produce no strains beyond safe limits in the extreme fibers of those posts. It involves calculations of a complicated and abstruse character, but it is possible to design a structure in this way which is perfectly secure. Mr. Morison.

The frames on many tall office buildings have been designed on this principle, though in many instances the calculations have been of the most crude and careless kind. The portal of a through bridge is really a construction of the same order. This principle, however, has generally been used only in those places where there were serious objections to transverse diagonal bracing. The general practice indicates that there are good reasons why engineers have always preferred diagonal bracing when there were opportunities for putting it in. These reasons are of two kinds: First, with diagonal bracing and a base of sufficient width to prevent tension in anchorages, there is never any reversal of strains in the posts, and, furthermore, the strains caused by wind alone when distributed over the whole cross-section of the post produce very small unit stresses and very slight changes in lengths of posts. In the present design reversals of strains are likely to occur in the extreme fibers of almost every part of each post, and these involve an appreciable change in the form of the post. The second reason is that the stiffness of a beam or truss is in direct proportion to its depth, and, with the opportunity of giving a trestle bent a stiffness based on the whole width of its base, it does not seem right to accept a stiffness based only on the width of the post.

Bridge specifications, many of which are drawn with great care,

Mr. Morison. generally have the defect of specifying strength and not providing for stiffness. Under such specifications two structures can be accepted as equally good, though, under a load, one of them will have twice the distortion that the other will have. For immediate safety, strength is all that is needed; for long life of structure, stiffness is at least equally important. This objection may be raised to some special forms of trusses, favored because of their economy of material, in which much greater variations occur in the strains of individual members than in differently designed trusses containing a little more metal. As compared with a viaduct of the usual design, everything would seem to indicate that the transverse deflections under strain, and even the distortion of the posts under compression, would be large in the new Kinzua Viaduct. If this be so, it is not probable that this design will be repeated. The objection to diagonal rods in the panels of such a viaduct, with pin and turn-buckle connections, is perhaps more theoretical than real, and, in a structure with the rigid and excellent details of the one described, they certainly could do no harm. It would be an interesting thing to put such rods in one of the tall towers of this viaduct, and then compare the stiffness of this tower and the adjoining tower whenever an extraordinary storm passes through the valley.

Mr. Buck. R. S. Buck, M. Am. Soc. C. E.—The Kinzua Viaduct is one of the best known and most unique railroad bridges in America, and its reconstruction will doubtless command general attention, both at home and abroad. This paper and the articles appearing in the leading engineering journals, while not furnishing data adequate to form a conclusive verdict as to the merits or demerits of the design, furnish sufficient ground for discussion of certain of its salient features. It is quite possible that a more thorough treatment of the subject by those fully acquainted with the design may disarm some of the following adverse criticisms; but the speaker believes that the showing on which he is compelled to base his discussion fully justifies its character.

The author does not, in the speaker's opinion, furnish such full explanation of his methods of calculation, nor as many details, as the radical character of the design demands.

The paper leaves so much to the ingenuity and imagination of the reader that it is as difficult to reach a full understanding of his methods and deductions as it is to find full justification for the design. The calculations are involved, and appear to be contradictory. The reader is subjected to much unnecessary trouble by the absence of a clear statement of the meaning of terms in the equations, limits of integration, and the steps by which the several expressions on pages 34 and 35 are reached.

The speaker has been unable, despite persistent effort and some able outside assistance, to follow the calculations to a finish, but some points stand out in a measure independent of the statement of the general scheme, and can be judged as correct or otherwise.

Mr. Buck.

On page 29 appears the statement:

"The problem can also be stated to consist of finding the locations of the points of contraflexure, because, if these locations were known, it would only be necessary to resolve the resultant," etc.

This refers to the analytical treatment, but, beyond this statement, there is nothing discernible in the text, table of equations or stress diagrams, to indicate that any effort has been made by the author to find the true points of contraflexure or to place them anywhere else than arbitrarily at the centers of the members.

Referring to the graphical treatment on the stress diagram, Fig. 8, is the note: "The computation of wind stresses assumes the point of contraflexure at the middle of the column sections, portals and headers."

These two quoted statements conflict, and should be further explained, for, until they are, there is no logical sequence in what follows. It is quite possible that the points of contraflexure are not very far from the centers of the members in all cases except in the case of the bottom sections of the columns. Here, the assumption is materially erroneous, and the effect on the moments very considerable.

In this case, the author not only assumes the column to be absolutely fixed at its foot, but also apparently ignores the bottom strut as resisting bending at this point. As a matter of fact, the fixity of the column at its foot is almost entirely dependent upon this strut.

The only other features that tend to fix it are the two 1½-in. anchor bolts very near the center of the base, and the direct pressure on the column acting on its square end.

The anchor bolts, intended to resist only 12 000 lbs. direct tension, can do but little to resist bending in the column. With rollers under the leeward column and with direct pressure removed by up-lift on the windward column, due to the assumed wind load on the unloaded structure, the fixity of the columns due to direct weight is very uncertain.

In the opinion of the speaker, the conditions do not justify the assumption of more fixity at the bottom of the columns than that furnished by the bottom strut. The correct location of the point of contraflexure of the bottom section is not far above the foot of the column, instead of at the center. The result of this is to greatly increase the bending at the level of Strut 1 over what is shown in the author's results, and also to increase the up-lift on the windward column enough to make it exceed the dead-load stress, with the assumed wind load on the unloaded structure.

The speaker would like to know if the author means, by the closing paragraph of his paper, that if the moments of inertia of a group of members were doubled it would not materially increase the stiffness of the bent? Also, how many members he considers as "a group"?

Mr. Buck. The failure of the author to give the essential details of connections bearing on the peculiar features of this design renders it necessary, in order to even approach an adequate understanding of the subject, to have recourse to the articles on the same subject in the technical journals.* These articles appear to be based on data from an authentic source, and to bear the stamp of official sanction. It is therefore deemed proper to refer to these articles for information essential to an adequate discussion of the subject, but lacking in the paper. It is to be hoped that the author will furnish all these details, in order to render the paper complete within itself and to correct any error which may arise from having to go outside of the paper for information.

Assuming that the author's methods of calculation are reasonably approximate—which does not seem warranted until they are rendered logical and free from obscurity—there are still, in the opinion of the speaker, faults in the design. He believes it is a mistake to reject the time-honored diagonal bracing in a structure of such proportions, and to substitute a form of bracing which, even when most carefully and generously detailed, could hardly be made both sure and economical. In all bridges, outside of a suspension bridge, rigidity is one of the prime objects to be attained, and especially in the case of railroad bridges. Substituting for direct stresses bending in members, being themselves certainly not possessed of a high degree of rigidity, is depriving the concrete structure of rigidity to an extent that seems questionable.

The height of this viaduct, at its highest point, is 285 ft. from masonry to base of rail. Each tower is independent, and can receive its maximum wind stress with wind over only about 150 ft. longitudinally of the structure. Within this narrow limit wind strikes in gusts and with considerable impact. In a tower so high, where 54% of the wind pressure is applied at the top when loaded, and 34% when unloaded, where the dead load is not sufficient to prevent up-lift on the windward column with assumed wind on the unloaded structure, the dynamic effect of vibration is a consideration that should be provided for. In a design of this kind, the amount of lateral deflection of the towers under wind load should be carefully ascertained, and, although the speaker has made no calculation for this purpose, he deems it safe to venture the statement that it is much greater in this design than in the case where the ordinary form of bracing is used.

It would be a matter of great interest to know, from transit observations, just what the present structure deflects laterally under high wind.

The detailing of the lateral strut connections with the columns seems to the speaker hardly in keeping with the heroic and uncertain

* *Railroad Gazette*, Nov. 30th, 1900, and *The Engineering Record*, Dec. 1st, 1900.

service they have to perform. The use of clip angles to reinforce the attachment of the strut chords to the columns, applied so as to be almost wholly ineffectual, is not warranted by what appears to be the only consideration for so doing, namely, the saving of some weight in the gusset plates at these points. Mr. Buck.

The light character of the knee braces, and their method of attachment to the struts, seems to indicate that their function is ornamental rather than useful; yet, with sufficient metal and adequate attachment to the columns and struts, they could be made to add very materially to the strength and rigidity of the structure.

A study of these details goes far to explain the material saving in weight with which the author credits this design, for they bear evidence of scaling down the metal to an extent that can hardly be generally endorsed.

There is another feature of this design which, independent of the character of the lateral bracing, seems to lack due strength, namely, the 1½-in. anchor bolts which, having served in the old structure, now serve in the new. In the old structure it is understood that there were no rollers under either column, temperature movements being taken care of by slotting transversely the anchor bolt holes in the column shoes, thus permitting the shoes to slide on top of the piers. The bottom strut was probably bolted fast to the center pier. With the bridge unloaded there was a wind pressure of about 45 000 lbs. on each tower, or 22 500 lbs. on each bent, under the assumed wind loads on the unloaded structure. To resist this shear at the top of the piers, there being up-lift on the windward column, there was friction under the leeward shoe, the attachment at the top of the center pier and the shear of probably one pair of anchor bolts in the column piers. These connections, though somewhat uncertain because of the slotted holes, were doubtless adequate, especially as the friction under the leeward shoe alone was perhaps equal to the wind shear.

In the case of the new structure, the service demanded of these anchor bolts is much more severe. The shear is 55 600 lbs. to the bent, 2½ times that in the old bridge. Rollers are placed under the column to leeward of the prevailing heavy gales, and there is therefore practically no frictional resistance at the foot of this column, and, the anchor bolt holes being slotted 6 ins. on this shoe, the bolts can take no shear from the shoe under normal conditions. It is also reasonable to assume that the anchor bolt holes in the bottom strut at the center pier were slotted. With the assumed wind load on the unloaded structure there is, as before stated, up-lift on the windward column. Therefore, with practically no friction under the shoe of this column the whole 55 600 lbs. of the wind shear has to be taken up by the two 1½-in. anchor bolts in the windward pier. This seems to be more than should be exacted of such bolts, and it is difficult to understand how

Mr. Buck. the use of the old bolts, without reinforcement, could be justified where the wind shear was so greatly increased and the means of resisting it so materially reduced. In short, the factor of safety of the anchorage of bents to piers is reduced at least 2½ times, and, in addition, by the specific amount of friction lost under the leeward shoe.

The speaker fails to see where any advantage is gained by placing the rollers on the side away from the direction of prevailing heavy winds, as the capacity of the towers to resist the shear due to such winds is thereby so greatly reduced.

The bolts in the movable shoe should be just as capable of resisting direct tension from up-lift as those in the fixed shoe, and there would be no loss of friction, but, in fact, a gain, when the fixed shoe is to leeward.

Mr. Moisseiff. L. S. MOISSEIFF, Assoc. M. Am. Soc. C. E.—This paper offers so many novel features and distinct departures from the usual design of high viaducts in the best modern American practice that the discussion of its merits or defects should prove to be of considerable interest to the profession.

While designed according to the rather conservative specifications of the Erie Railroad Company, the design of the high towers denotes a radical departure from the type used heretofore, in omitting the sway diagonals. The bending moments caused by the assumed wind loads are intended to be resisted by the increased stiffness of the columns and horizontal struts which are strengthened by brackets. Not that this method of providing for wind stresses is entirely unknown or untried; it has been used very frequently, and with some reluctance, for sway bracing of through bridges, for wind bracing of high steel-skeleton structures, and in all portals. But, in all cases where sway diagonals have been omitted, this omission has been due to lack of head room or due to wall space taken up by windows and doors, and was thus made compulsory by the necessities of the case.

It seems so much to be almost an axiom of good structural design never to resist by bending stresses what can be done as well by direct axial stress, that the application of the "portal bracing" principle in a high viaduct, subject to high wind pressures and having no limitations of head room, is surprising.

Every engineer who has had to design trusses and frames of great magnitude, and containing long members which require bracing and intermediate supports, knows the great amount of material required by these secondary members and how rapidly the percentage of material required to attain the practical design of the truss increases in comparison with the stress volume. Every attempt to reduce the cost of these secondary members is a step in the right direction; but it seems doubtful whether the saving in material claimed by the author for his design has been obtained without impairing the efficiency of

the structure to resist lateral forces and vibrations, especially the latter. Mr. Moisseiff.

While a comparison of the tower bracing as designed with the usual diagonal bracing of high trestles and towers with stiff members will certainly show a saving in weight for the portal-braced type, a comparison of their relative stiffness will be greatly in favor of the one with diagonal braces. A high trestle bent of the type used by the author is certainly not stiffer than the old tie-rod braced bent. In order to get a fair comparison between the portal-braced system and the system with diagonals, the former should be compared with a bent braced by tie rods. The speaker believes that the claimed economy will then soon vanish. The bending moments induced by wind pressures may be provided for by the stiff and deep struts just as well as by rigid diagonals, but the vibrations caused by them will be considerably higher than in diagonally braced towers. The tower will, in fact, have some similarity to a tuning fork.

The author deserves credit for the application of modern methods to the analysis of the stresses. The principle of least work used by him is a handy and convenient tool for the determination of stresses in frames, and especially for the study of new types of trusses. The value of the author's analytic treatment would be greatly enhanced if the notation used and the processes gone through were stated more explicitly and carefully.

In his analysis the author assumes each frame to be fixed rigidly at the bottom. This assumption is erroneous. The two old wrought-iron bolts, of $1\frac{1}{2}$ ins. diameter and some 18 ins. apart, are not sufficient to insure the necessary degree of fixedness to the columns. Besides, the high bents have one leg on roller bearings, which is certainly not very conducive to the rigidity of the lower column end. But this assumption of fixed ends for the lower ends of the columns is a vital and extremely important factor in the author's theory. These ends are only in so far fixed as the bottom horizontal struts provide for, and not much more.

Instead of considering for a one-story bent, as the author does, the upper horizontal beam only, the correct way would be to take also the lower horizontal strut into account, and include its work of deformation in the general formula. The same idea should be followed through for all bents, always including the bottom horizontal strut as an essential member.

For a single-story bent the problem is quite simple. In view of the general application of this case for sway bracing and portals of bridges, a simple formula to find the points of contra-flexure of the columns will be given here which does not necessitate in its use the application of the higher calculus, and is rather handy.

Professor Winkler* deduces the same formula, and the following is

* "Querkonstructionen der Eisernen Brücken."

Mr. Moisseiff. a simplified deduction thereof by the principle of least work, as used by Mr. Grimm.

The work done by any beam subject to bending, in assuming its new position of equilibrium, is given by

$$A = \int \frac{M^2}{2\,E\,I}\,d\,x,$$

neglecting the work done by the axial force. E is here the modulus of elasticity, and I the moment of inertia of the beam in flexure. If the beam is fixed at both ends, Fig. 10, and is of length l, then the moment at any point x is

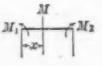


FIG. 10.

$$M = M_1 + (M_2 - M_1)\,\frac{x}{l}.$$

In order to find the work done by the beam in attaining its new equilibrium, we substitute this value in the foregoing work formula:

$$M^2 = M_1^2 + (M_2 - M_1)^2\,\frac{x^2}{l^2} + 2\,M_1\,(M_2 - M_1)\,\frac{x}{l}$$

$$A = \int \frac{M^2\,d\,x}{2\,E\,I} = \frac{1}{2\,E\,I}\int_o^l \left[M_1^2 + (M_2 - M_1)^2\,\frac{x^2}{l^2} + 2\,M_1\,(M_2 - M_1)\,\frac{x}{l}\right] d\,x = \frac{l}{6\,E\,I}\,[M_2^2 + M_1^2 + M_1\,M_2]$$

This holds true for any beam fixed at both ends by the moments M_1, M_2. This being given, the one-story bent can now be treated.

Fig 11 represents a frame of height h, and horizontal struts with fixed joints subject to the action of a horizontal force $2\,P$ at the joint 1. Let $O\,O$ denote the line of contraflexure, and h_1 and h_2 the distances from the points of contraflexure to the upper and lower struts, respectively. The total amount of deformation work caused by $2\,P$ consists of the sum of the work done by its four individual members.

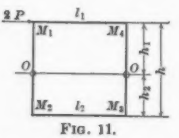


FIG. 11.

$$A = \frac{1}{6\,E}\left[\frac{h}{I}\,(M_1^2 + M_1\,M_2 + M_2^2) + \frac{l_2}{I_2}\,(M_2^2 + M_2\,M_3 + M_3^2) + \frac{h}{I}\,(M_3^2 + M_3\,M_4 + M_4^2) + \frac{l_1}{I_1}\,(M_4^2 + M_4\,M_1 + M_1^2)\right]$$

where I is the moment of inertia of the columns I_1 of the upper strut, and I_2 of the lower strut. For equilibrium, the work done will be a minimum.

The moments are:

$$M_1 = Ph;\ M_2 = Ph_2 = P\,(h - h_1) = Ph - M_1;\ M_2 = -\,M_2;\ M_4 = -\,M$$

Differentiating partially with respect to h_1 as the independent variable:

$$\frac{\delta\,M_1}{\delta\,h_1} = P;\ \frac{\delta\,M_2}{\delta\,h_1} = -\,P;\ \frac{\delta\,M_3}{\delta\,h_1} = +\,P;\ \frac{\delta\,M_4}{\delta\,h_1} = -\,P.$$

Mr. Moisseiff.

6 E and P, as constants, divide out, and we get:

$$\frac{A}{\delta h_1} = 0 = \left[\frac{h}{I}(M_1 - M_2) + \frac{l_2}{I_2}(M_3 - M_2) + \frac{h}{I}(M_3 M_4) + \frac{l_1}{I_1}(M_1 - M_4)\right]$$

Introducing the values set for the moments:

$$\frac{h}{I}(2\,h_1 - h) + \frac{l_2}{I_2}(h_1 - h) + \frac{l_1}{I_1}h_1 = 0,$$

whence,

$$h_1 = h\,\frac{\frac{h}{I} + \frac{l_2}{I_2}}{\frac{2\,h}{I} + \frac{l_2}{I_2} + \frac{l_1}{I_1}}$$

This formula gives the position of the point of contraflexure, and with it the moments at the four joints are readily found. It will be noted that the position of the points of contraflexure is independent of the external force, and that it is only a function of the moments of inertia of the members and their lengths. This might have been expected to be so.

For all the members of the same moment of inertia, and for the horizontal struts of the same lengths, $I_1 = I_2 = I$; $l_1 = l_2$, we find

$$h_1 = \frac{h}{2}$$

The point of contraflexure is at the middle of the column. Thus we find that the assumption that the point of contraflexure is at half the height of the columns is true only for a rectangle with sides having the same moment of inertia.

Applying the above contraflexure formula to the one-story bent taken by the author, the point of contraflexure will be almost 21.36 ft.

From the general equation of moments, as given by the author,

$$M = P_5\,z - X_5\,(h + n\,z).$$

For the point of contraflexure $M = P_5\,h_1 - X_5\,(h + n\,h_1) = 0$.

Whence

$$X_5 = \frac{P_5\,h_1}{h + h_1\,n} = \frac{6.7 \times 21.36}{4.765 + \frac{21.36}{6}} = 17.2 \text{ tons.}$$

The greatest moment at Joint 1, $M_1 = -\,X_5 \times 4.765 = -\,81.85$ foot-tons.

$$M_2 = 37.6 \text{ foot-tons.}$$

A comparison of the author's figures shows the effect of his assumption of fixed ends. The point of contraflexure will naturally be farthest from the stiffer joint, and it actually moves down and thereby increases the moment on top by some 14 per cent. Of course, the lower bents, as designed by the author, are so massive that they offer more than ample resistance to any increased moments. But this is due to the allowed low working stress.

Mr. Moisseiff. The lower horizontal strut is only 30 ins. deep, and its moment of inertia is less than one-twentieth of that of the upper 8-ft. deep strut. The point of contraflexure will, of course, move nearer to the lower end, and, making all due allowance for the stiffening effect of the brackets, it will be found to be 0.87 of the height of the lower story from the 8-ft. strut, or 52.7 ft. The sum of all the X-forces above the point of contraflexure, which is but the transferred wind load, will be 52.5 tons, and the maximum moment in the column will be 192.5 foot-tons, instead of 108.54 foot-tons as given by the author, or some 78% higher. The extreme fiber stress due to this moment in the columns is 7 000 lbs. per square inch.

Taking the stresses found by the author for dead, live, and wind load and omitting the traction stresses, and adding to them the extreme fiber stress due to bending, the greatest stress will reach 15 500 lbs. per square inch. This exceeds by 11% the maximum stress of 14 000 lbs. given by the author. Including traction, the stress will be 20 000 lbs.

It would appear from the paper that the lower bents are simply cut-off upper portions of the high bent, so that, for instance, in the four-story bent the 8-ft. horizontal strut forms the lowest member. If this is so, and if the author appreciated the great stiffening effect of the lower strut, why did he not put one as deep or deeper in the five-story bent, instead of the comparatively flimsy 30-in. strut? And if he deemed the bottom horizontal struts of no importance, as it results from his theory, then why did he not replace these deep bottom struts by the light ones used in the highest bent, and save material thereby?

Mr. LaChicotte. H. A. La Chicotte, M. Am. Soc. C. E.—The principal feature of this paper is the method of calculating the stresses in the structure, although if the title of a paper is an index of the contents, it is to the structure itself that the author would direct particular attention. That part of the paper devoted to a description and illustration of the work is so limited and devoid of detail as to necessitate going to outside sources for information relative to this structure sufficient to render an intelligent discussion of the subject possible. Fortunately, timely articles, appearing in recent issues of two technical journals, supply many of the necessary data which should have been given by the author.

The reasons offered for adopting such a design as the author submits for criticism are by no means satisfactory, and it is to be hoped that discussion will bring out the relative merits of the several designs referred to, as well as a properly executed design of the articulated type. While the latter type was excluded from competition, it would no doubt possess advantages over that used, both in economy and rigidity. It is significant that the weight of the new structure should be nearly double that of the old, even after making due allowance for the increased traffic.

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The fact that the columns in a bent approach so nearly together at the top is of vital importance in the adopted design; and, without the verification of actual figures, the use of this type for a double-track structure would be precluded by reason of the greatly increased bending moments in the tower bents, and the resulting increased cost.

It is not necessary to state that the most suitable design for any structure is that which most nearly fulfills all the requirements of each particular case; and while allowance may reasonably be made for the personality of the designer, where matters of judgment and taste are involved, it is nevertheless incumbent upon him to adhere strictly to the principle of utility and suitability of design. This principle, when broadly interpreted, will be found to be amply elastic to cover all practical cases. Originality and fertility of resource are always commendable and usually valuable, but, unless guided by good judgment and practical common sense, are apt to lead to extremes. Neither prejudice against established forms, nor a liking for the unique may be allowed to dictate what a design should or should not be.

In the present case there are several details of construction open to criticism, and, while the speaker does not wish to be understood as condemning the design as a whole, there is nothing in the paper under discussion, nor in the requirements of the case, so far as known to him, which, in his opinion, would justify its adoption.

As to the analytical treatment for the determination of wind stresses, the author has presumed too much and left so much unexplained that to read the paper intelligently means to almost work out the problem on independent lines.

In fact, it was not until the speaker had worked out the problem for himself that he realized wherein lay the difficulty with the author's method.

Why the author felt justified in considering the columns as fixed at the base is past comprehension; and he is clearly in error in so doing. In the case of a one-story bent, not only is the base of the tower leg not fixed, but very much less fixed than the top, and the point of contraflexure in the column should be somewhat below the middle—that is, nearer to the smaller moment tending to fix the ends.

An incongruous result of the author's assumption and treatment is that the point of contraflexure is nearer the greater moment assumed as applied to the base of the column to insure its absolute fixity.

As a matter of fact, in applying the principle of least work, it should be made to cover the strut connecting the bases of the columns, as well as the other members of the bent, as little or no reliance can be placed in the anchor bolts as fixing the legs—especially on the expansion side of the tower.

The moments in the lower struts may always be expressed in terms of the moment of the adjacent column. If, in the expression for the

Mr. LaChicotte.

moment of the column, $M = P_5 z - X_5 (h + n z)$, z is put equal to a, there results, $M_a = 208.77 - 9.96\ X_5$, for the moment in the strut when the sign of M is changed.

This moment divided by the half width of the tower base gives

$$X_4 = \frac{M}{i} = 20.96 - X_5.$$

Whence

$$\int M \frac{\delta M}{\delta X_5} dx = -\left[20.96 - X_5\right]\frac{x^3}{3}\Bigg]_{x=o}^{x=9.96} = -6972.69 + 332.67\ X_5.$$

This result, divided by the moment of inertia of the strut, and added to the equation of condition already found for the other members of the bent gives,

$$\frac{36.06\ X_5}{107500} - \frac{26760.27 - 1758.54\ X_5}{10750} - \frac{6972.69 - 332.67\ X_5}{5511} = 0$$

Whence, $X_5 = 16.8$, $X_4 = 20.96 - X_5 = 4.16$.

The resulting moments (Fig. 12) on the bent are:

At top of Column $M_5 = 80.1$;

At base of Column $M_4 = 41.43$.

The point of contraflexure is located 20.6 ft. from the top of the column, or, about 5 ft. from the theoretical middle point assumed in the graphical solution.

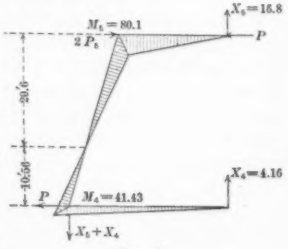

FIG. 12.

Proceeding in a similar manner in the case of a two-story bent, the unknown force acting at the middle of the strut uniting the bases of the columns is found to be

$$X_3 = 34.363 - (X_5 + X_4),$$

the equation of moments $M = -[34.363 - (X_5 + X_4)]\ x$,

from which there results:

$$\int M \frac{\delta M}{\delta X_5} dx = -\left[(34.363 - [X_5 + X_4])\right]\frac{x^3}{3}\Bigg]_{x=o}^{x=20.3} = -95\,820.49 + 2\,788.48\ (X_5 + X_4)$$

also,

$$\int M \frac{\delta M}{\delta X_4} dx = -\left[(34.363 - [X_5 + X_4])\right]\frac{x^3}{3}\Bigg]_{x=o}^{x=20.3} = -95\,820.49 + 2\,788.48\ (X_5 + X_4).$$

These expressions, when divided by the moment of inertia of the lower strut, must be added to the equations of condition already

found by the author. Performing the necessary operations, there result the following equations: Mr. LaChicotte.

$$182\,930\ X_5 + 165\,309\ X_4 = 5\,400\,509;$$
$$25\,668\ X_5 + 26\,666\ X_4 = 797\,015.$$

Whence, $X_5 = 19.29$, and $X_4 = 11.3$;

also, $X_3 = 34.363 - (X_5 + X_4) = 3.77$.

The resulting moments on the several members of the bent, as well as the positions of the points of contraflexure, are shown in Fig. 13.

These results do not differ materially from those of the author, but the resulting moments on the columns are uniformly greater at the top and less at the base, owing to the less degree of fixity at the base than was assumed by the author. It is noticeable also that the point of contraflexure in the upper column of a two-story bent is considerably lower (nearly 6 ft.) than in a one-story bent.

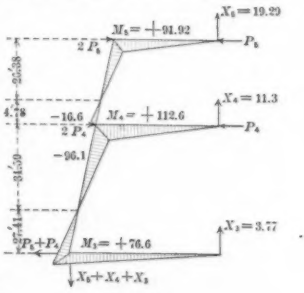

FIG. 13.

The position of the point of contraflexure depends upon the relative fixity of the ends of the individual member.

Owing to the symmetry of the bent, the ends of the struts are equally fixed, and the points of contraflexure are at their middles. The ends of the columns, however, are not equally fixed, as a rule, and the points of contraflexure will be found nearer the smaller moment.

If the struts at the bases of all the smaller towers are those used at the corresponding panel points of the large tower, counting from the top, no very serious errors will be found in the stresses of the smaller towers; but if, as in the case of the large tower, the bottom struts have been made slender as compared with those above, the resulting stresses, when these struts are included in the system, will be found to differ substantially from those given by the author.

The erroneous assumption of fixity at the base of the columns, and the failure to extend the calculations to the bottom struts, have led to serious errors in the large tower.

Here, the bottom strut has a moment of inertia of 2 216 quad. ins., while the half length is 51.02 ft. The moment at the base of the column is:

$$2\,704.892 - 51.02\,(X_5 + X_4 + X_3 + X_2 + X_1 + X_0)$$

whence, $$X_0 = \frac{2\,704.892 - 51.02\,(X_5 + X_4 + X_3 + X_2 + X_1)}{51.02}$$

Mr. LaChicotte. which must be taken negatively, as it produces a left-hand or negative moment; hence,

$$-X_0 = -53.016307 + (X_5 + X_4 + X_3 + X_2 + X_1) \ldots\ldots\ldots\ldots (a)$$

$$M = -X_0 x; \quad \frac{\delta M}{\delta X_0} = -x; \quad \int_{x=0}^{x=51.02} M \frac{\delta M}{\delta X_0} \delta x = 44\,269\, X_0.$$

Substituting the value of X_0 from (a) there results:

$$\int_{x=0}^{x=51.02} M \frac{\delta M}{\delta X_0} \delta x = -2\,346\,979 + 44\,269\,(X_5 + X_4 + X_3 + X_2 + X_1)$$

The value of this integral, when divided by the moment of inertia of the lower strut, must be added to each equation of condition. Multiplying each equation by 107 500, the value of the largest moment of inertia, and grouping the values of X_5, X_4, X_3, etc., there result the following five equations, containing five unknown quantities:

$$4\,530\,396\, X_5 + 4\,512\,770\, X_4 + 4\,365\,420\, X_3 + 3\,957\,800\, X_2 + 3\,361\,799\, X_1 = 222\,736\,629.$$
$$4\,512\,770\, X_5 + 4\,519\,221\, X_4 + 4\,365\,420\, X_3 + 3\,957\,800\, X_2 + 3\,261\,799\, X_1 = 222\,469\,029.$$
$$4\,365\,420\, X_5 + 4\,365\,420\, X_4 + 4\,383\,375\, X_3 + 3\,957\,800\, X_2 + 3\,261\,799\, X_1 = 217\,953\,209.$$
$$3\,957\,800\, X_5 + 3\,957\,800\, X_4 + 3\,957\,800\, X_3 + 3\,989\,471\, X_2 + 3\,261\,799\, X_1 = 202\,011\,089.$$
$$3\,251\,799\, X_5 + 3\,261\,799\, X_4 + 3\,261\,799\, X_3 + 3\,261\,799\, X_2 + 3\,318\,056\, X_1 = 170\,430\,318.$$

The values of the unknown quantities resulting are, $X_5 = 19.454$, $X_4 = 11.673$, $X_3 = 8.138$, $X_2 = 6.494$, $X_1 = 6.528$.

The value of X_0 is found from Equation (a), and is $X_0 = 0.729$.

The resulting moment, and positions of the points of contraflexure, are shown in Fig. 14, the notation of the author having been followed, in order to facilitate comparison with the results given by him. The effect of the author's assumption is strikingly illustrated in the case of the columns of the first story, where the moments at the top are increased about 70%, and in Strut No. 1 where there is an increase in the moment of about 32 per cent.

It is to be noticed also that the point of contraflexure in the column of the first story is to be found about 20 ft. below the middle point, while the moments given by the author show it about 4 ft. above the middle. This illustrates the danger of arbitrary assumptions as a basis of computations, and points clearly to the duty of using simple determinate forms unless conditions are such as to require otherwise.

The increased moments in the first story will run the unit stresses considerably above the limits fixed by the author, and would seem to necessitate the use of diagonal rods, or lower struts of greatly increased moments of inertia, to correct the evil.

It will probably be noticed that while the author has expressed the moments in foot-tons, the moments of inertia of the several members have been expressed in quad. inches. This is a very fruitful source of

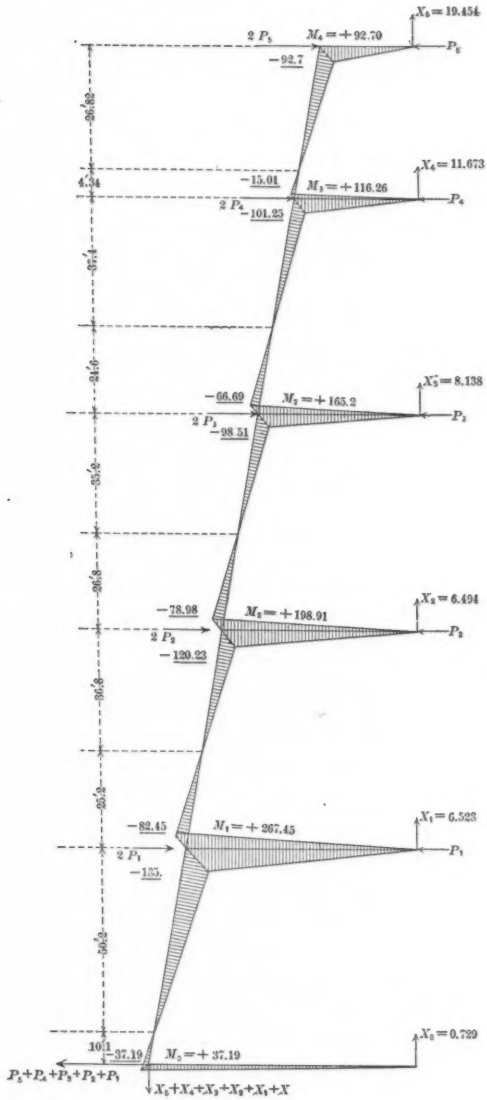
$X_6 = 19.454$
$2\,P_5$
$M_6 = +92.70$
P_6
-92.7
26.82
$X_4 = 11.673$
4.34
-15.01
$M_5 = +116.26$
P_5
$2\,P_4$
-101.25
37.4
24.6
$X_3 = 8.138$
-66.69
$M_3 = +165.2$
P_3
$2\,P_3$
-98.51
35.2
26.8
$X_2 = 6.494$
-78.98
$M_2 = +198.91$
P_2
$2\,P_2$
-120.23
36.8
25.2
$X_1 = 6.523$
-82.45
$M_1 = +267.45$
P_1
$2\,P_1$
$-135.$
50.2
10.1
$X_0 = 0.729$
-37.19
$M_0 = +37.19$
$P_0+P_4+P_3+P_2+P_1$
$X_0+X_4+X_3+X_2+X_1+X$

FIG. 14.

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error in ordinary computations of deflections, but in the present case no error results because it is the relative and not the actual values of the several moments of inertia which affect the results.

The work of deformation in a truss or beam may be produced by a moment, direct stress, shear or torsion, in any combination, depending upon the manner of application of the external forces.

In ordinary bridges torsion is avoided or reduced to a minimum; so that its effect may generally be neglected. In trusses the shear is usually replaced by direct or axial stresses, and in beams the work of deformation due to shear is very small as compared with that due to direct stress or bending; so that the effect of shear, as such, may usually be neglected without sensible error.

Under ordinary conditions, the only forces that need be considered as producing deformation are those of bending and direct stress; so that the work of deformation, which must be a minimum, may generally be expressed by the formula

$$W = \int \frac{M^2\, dx}{2\, E\, I} + \int \frac{P^2\, dx}{2\, E\, A} = \text{Min.}$$

Where M = the moment of the external forces;
E = the modulus of elasticity;
I = the moment of inertia of the section;
P = the axial or direct stress acting upon the section the area of which is A;
and dx = the element of length.

It is a matter of common knowledge that a force acting on a bridge member of ordinary proportions will distort it much more by bending than by either direct tension or compression, and it requires more material to resist the action of bending than that of direct stress. This is but another way of saying that the work of deformation of an axial force is very small as compared with that of a moment, and the author properly recognized this fact in dropping from the foregoing equation the expression for the work of the direct stresses, reducing the equation to the form

$$W = \int \frac{M^2\, dx}{2\, E\, I} = \text{Min.}$$

But in so doing he must have realized fully that by omitting the diagonal bracing of the towers it became necessary to introduce into the tower legs to resist the bending, a sectional area many times that of the discarded diagonals. Nearly one-half the sectional area of the tower legs is required to resist bending, while a small part of this additional area would suffice for an effective system of diagonal rods.

The speaker fully realizes that the lighter structure is not necessarily the cheaper, as there are many other considerations besides that of weight of material. Facility of erection is important, but the data

given by the author do not point to undue economy in this respect, even after allowing for the removal of the old structure. Uniformity of details has an important bearing upon the cost, both of manufacture and erection; but uniformity is very often secured by too great a sacrifice of material, and the ultimate cost of the structure thereby increased rather than reduced. If the contractors furnished the work in this instance at a pound rate, they are to be congratulated for devising a type of structure involving a minimum shop cost and a maximum tonnage. Mr. LaChicotte.

GUSTAV LINDENTHAL, M. Am. Soc. C. E. (by letter).—The first Kinzua Railroad Viaduct (built of wrought iron) attracted world-wide attention from engineers at the time of its construction (1880–81), because of its great height and the rapidity of its erection. Its articulated design of short spans—expansion joints at every tower, and the towers made up of braced bents, which were subject to large bending strains in the direction of the track—was not favorably thought of at that time by some engineers, who believed that the rigidity and durability of the viaduct were impaired thereby. Experience has proved the prediction to be correct. It is well known that, almost from the beginning, trains over that structure were obliged to slow down to about 5 miles per hour. Even then the vibrations, although not dangerous, caused unpleasant sensations. Mr. Lindenthal.

The old structure was typical of the American form, as distinguished from the European form, of metallic viaducts. In the latter the aim was always to obtain great rigidity, so that trains could pass over them without diminution of speed. Compact towers, continuous girders and longer spans were the leading features. Long-span arches also occur as parts of such viaducts over deep ravines, as in the Garabit Viaduct, in France, and the Münster Viaduct, in Germany.

It is a singular fact that in American bridge specifications, up to this date, as far as known to the writer, there are no provisions insisting upon rigidity of structure. If obtained, it is accidental, or the result of extra provisional care by the designing engineer, which he is under no obligation to give. The old Kinzua Viaduct complied in all respects with the old Erie specifications, which were then considered among the best. Nevertheless, it was a swaying structure, unfit for trains at ordinary speeds, and inferior to similar structures in Europe, built ten and twenty years earlier.

Among the earliest was the well-known double-track viaduct at Freiburg, in Switzerland (built in 1862), about 225 ft. high and 1 100 ft. long. The Iglava Viaduct in Austria, the Ossingen Viaduct in Switzerland, and others of considerable height and length, followed the Freiburg example. Faulty as some of their details are, in the light of present engineering knowledge, the essential fact remains that even at that early day it was found practicable to build high iron viaducts

Mr. Lindenthal. of sufficient rigidity to carry trains without diminution of speed. The weight of locomotives and cars on these viaducts since then has also increased, but they have stood wear and tear very well, and, so far as known, will, with few repairs, be serviceable yet for a number of years.

In the American construction, greater importance was attached to the temporary advantage of rapid erection, without falseworks, and with the fewest men, far away from the bridge shops. It led to a type of thin-legged and short-span trestle structures of flimsy appearance, inferior in rigidity to the European type. The trestle type may answer well enough for low heights, but is not suitable for greater heights, as proved by the old Kinzua Viaduct, and by a few other American railroad viaducts of less height which could be named.

The weight of locomotives and cars on the Erie Railroad has nearly doubled in the last twenty years, and the viaduct might have had to be rebuilt in any case; but, in doing so, the time had arrived also for a rigid structure over which trains could be run at full speed. This should have been the leading thought in the new design. It appears from this paper, and from the character of the plans, that such was not the case.

Because rolling stock has doubled in weight in the last twenty years, it does not follow that it will double again in the future. The limit will soon be reached (by reason of the crushing strength of hard steel in rails and wheel tires). An increase of 50% in the weight of the present heaviest rolling stock is improbable. But if it should occur, that margin is amply covered by the working stresses of the present Erie specifications. The new structure, under these conditions, should have been designed to be a permanent one, to last centuries.

For an intelligent judgment of the plans, Mr. Grimm does not furnish sufficient data. There is, for instance, no elevation or diagram of the viaduct, showing the arrangement and location of the expansion joints. The description of them on page 24 is not intelligible without such elevation. No weights of the new structure are given, from which the economy of design could be judged, in view of the author's allegation that the choice of the peculiar system of wind bracing in the towers was prompted by economy, when compared with other systems of bracing. Neither are data given as to the behavior of the structure under trains.

Some remarkable assumptions have been made. For instance, below the diagram (Fig. 8), showing the stresses in the tower bracing from braking, is a note, as follows:

"The computation of the stresses from the braking of trains assumes the point of contraflexure at half the height of the towers."

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From which it appears that the towers are assumed to have at the top bending moments of the same value as those at the bottom, where the tower legs are anchored. How these are possible with the shallow plate-girders of the superstructure (of different heights and discontinuous chords) requires an explanation.

The horizontal force from a braking train per tower is not stated on the diagram, but may be judged, from the strains given for the stiff web members, to be about 50 000 lbs. for one side, or 50 tons per tower. For prudential reasons, the entire height of the tower, and not half its height, must be taken as the leverage for that force. The placing of an expansion joint in the superstructure at every third or fifth tower has not the slightest influence on that condition. The bending moments at the base will, therefore, be double those assumed; and there can be no bending moments at the top of the towers worth considering. Whether the columns and anchorages at the foot of the high towers are sufficiently strong to resist those larger bending moments cannot be judged from the meager data in the paper.

There are the further questions: How much do the towers bend or deflect under the horizontal braking force applied at the top? And does not the bending of the towers subject the riveted connections (of the track girders) at their tops to large bending strains, for which these connections are not adapted? Would not the high towers bend so much under a suddenly braked train as to cause the girders to slip off the roller bearings at the expansion joints, and cause the viaduct to collapse? Is not the only safety against such an occurrence in the continuity of the track structure?

These important questions can be accurately answered only with the aid of deflection diagrams of the towers, which are not given.

The most characteristic feature of the towers, however, is in the peculiar lateral bracing, called ordinarily the wind bracing, because (except on curves) it is usually dimensioned only for wind pressure, assumed to act quiescently. But the fact cannot be overlooked that this bracing is also exposed to the lateral impact and vibrations generated by the moving trains. These strains are dynamic in their nature, and therefore more dangerous on that account. The impact and vibration increase as the square of the train speed. Every trackman knows from experience that on a shaky structure trains must run slowly, although he may not know the theoretical reasons for that necessity.

Most bridge specifications make allowance for vertical impact by the addition of arbitrary percentages to the train-loads. A similar allowance should be made for horizontal impact, for the lateral vibration of fast moving trains, and for the violently alternating strains on the bracing. In ordinary bridges, these strains may safely enough be assumed to be included in the wind strains; but in high viaducts, or long spans, such an assumption is not warranted by observed facts.

Mr. Lindenthal. In the new Kinzua Viaduct, the lateral bracing is without diagonals in the towers. Stiff gusset connections with the horizontal braces take their place, on the plea that they are more economical. This may well be doubted. Does that arrangement produce a more rigid construction laterally? It is obvious that a deflection diagram of the tower bents at right angles to the track would furnish the emphatic answer "No."

Higher mathematics, as applied to the principle of least work, was probably never used for a poorer purpose than in the calculation of this tower bracing, needlessly made to be statically indeterminate, and which is inferior to the statically determinate bracing in the old towers, unsubstantial as the latter had been. The whole procedure shows a misconception of the fundamental conditions of good designing, and theoretical trifling with a serious problem, involving the safety of the structure.

Its details are also open to question. For instance, the large shearing strains in the tower columns from the bending moments at the gusset connections are taken up by the lacing angles. The single shear rivets of these angles, therefore, have to resist strains additional to those of ordinary lacing. The alternating strains from the horizontal impact and vibration expose the lacing angles to a sawing action. There is great likelihood here of the rivets loosening and endangering the stability of the towers. By turning the columns one-quarter around, so that the solid web plates would have taken the shearing strains, that danger, at least from loose rivets, would have been avoided.

But, after all, the defects in the bracing may be readily cured by the insertion of additional triangular or diagonal braces. The necessity for them in the higher towers will arise sooner or later.

If the new viaduct should seem at present to be stiffer than the old one, it can be so only on the low bents and under slow trains, and by reason of the greater weight and bulk of the new structure.

A rigid viaduct does not necessarily mean a more expensive one. Expansion joints should be the fewest in number. The structure is only about 2 000 ft. long, and dilatation joints placed at each end of the viaduct would have been sufficient to take care of temperature changes. The towers should not be subjected to the heavy strains from braking trains; they are not adapted for them. Resistance to braking trains can be furnished without them. Looseness of structure, lengthwise with the track, was a potent cause of the shakiness of the old structure.

If a competition of designs and bids had been invited, on the condition that the new structure must be suitable for fast trains, and that the sudden braking, or the starting up, of a double-header train must in no wise endanger its safety, then the proof would most likely have

been furnished, in a variety of designs, of its being readily attainable, and at no greater cost (if as great) than the structure described in this paper. Mr. Lindenthal.

F. H. Cilley, Esq. (by letter).—This paper raises many most interesting questions, both theoretical and practical. Mr. Cilley.

For the calculations, Mr. Grimm gives preference to an application of what is known as "the principle of least work," a principle so little known and less understood, even by many who use it, that a brief examination and exposition of it here may be of interest.

As commonly stated, this principle declares that "the actual stresses in a structure are those which make its work of deformation a minimum." But, set forth in these terms, the principle is in several ways most misleading when not actually incorrect. It suggests that it is based on the popularly accepted law that all natural processes tend to pursue the course most economical of energy. Yet, as this latter is not a true law, the principle, in so far as it is correct, cannot be simply a special application of it.

If one sought to interpret the law, ignorant of the rules for its application, one's impression would probably be that it asserted that the actual stresses (which are, of course, in equilibrium) represented less stored energy than any other set of stresses (not balanced, and therefore momentary), that is, that it asserted that the energy of the frame in the state of equilibrium under given forces was less than in any non-equilibrated state under these forces—a law of dynamics.

But, actually, it in no way states conditions of equilibrium, and is not a law of dynamics. What it really asserts is that the set of stresses in equilibrium under the given applied forces, which would wholly disappear on the removal of those forces (that is, which would occur under the applied forces if there were no initial stress), is that one among all possible sets of stresses in equilibrium under the given applied forces (for there are numberless such sets for all indeterminate structures), for which the potential energy of the structure is least. And the reason for this fact is of the simplest. For structures obeying Hooke's law, the distortion accompanying the application of any given set of forces is independent of the existence of initial stress, and, therefore, not only the change in stress, but the work done to the structure by the applied forces, and, therefore, the change in the potential energy of the structure, are independent of the existence of initial stress. Thus, the total potential energy of a structure in equilibrium under given applied forces is its initial potential energy (due to its initial stresses) plus the (invariable) change of potential energy due to the application of the given forces. The total potential energy evidently is least when the initial potential energy is zero, that is, when there is no initial stress, and thence the law as last

Mr. Cilley. stated. It is not a law of equilibrium; it is not a special case of a great universal law of Nature, but is simply a limited law of the theory of elasticity, dependent for its truth, even in the second form given herein, on whether Hooke's law happens to hold, or does not happen to hold, for the materials and manner of distortion of the structure under consideration. In its first form it is not necessarily correct, even where Hook's law does hold, for the stresses it determines are, in general, simply the changes in stress due to the given applied forces, which are only the actual stresses when there chances to be no initial or self-balancing stresses.

Now, as to its application: First, the potential energy of the structure under the given applied forces must be expressed in terms of any variable set of stresses in equilibrium with them, and including in their variation the set of stresses sought. This is done by noting how many more stresses have to be determined than there are conditions provided by statics. As many of the stresses as have thus been found to be in excess are then designated by letters as unknowns. They evidently may be independently given any arbitrary values whatever, and the conditions provided by statics still be satisfied. They are, therefore, independent variables. The conditions of statics now permit all the stresses in the structure to be expressed in terms of the applied forces and these unknown stresses. Thence it is very easy to express the corresponding potential energy of the structure. Now, for the solution sought, the set of stresses for which there is no initial stress, or the change in stress under the applied forces—the set of values of the unknowns—the independent variables—must be found, which will make the expression for the potential energy of the structure a minimum. This is purely a question of mathematics. As is known, the differential coefficient of the expression for the potential energy of the structure, with respect to each of the independent variables, must be zero. This gives as many equations as there are independent variables. Stress being proportional to strain (Hooke's law), the expression for the potential energy of the structure is quadratic in terms of the variables. These partial differential coefficients are therefore linear. The values of the independent variables, consequently, are determined simply by solution of linear equations. Such is, in outline, the complete explanation of "the principle of least work" and its application.

What are its merits and demerits as a working method? it may be asked.

It may be used almost mechanically, and therefore saves thought, but at the expense of understanding. Like most integral theorems, it is a sort of blanket which hides the details and prevents the perception of the significance and purpose of the procedure. On this account, the writer is inclined to prefer the use of a more objective

method, in which the structure is conceived as being so cut up or otherwise altered as to make it statically determined, place forces and moments at the various cut sections or otherwise altered points to replace the restraints removed, then, preferably, by aid of the method of virtual work (also known as the method of virtual velocities or displacements), determine the corresponding displacements, the equation of which gives the conditions which must exist in the original uncut and unaltered structure. In the case of no initial stress this results in precisely the same linear equations as before, * but here the meaning of each is clear; one is in close touch with the physical problem at all times; errors are easily discovered and corrected; the consequences of changes are readily perceived, and one gains a really intimate insight into the workings of the structure which is the object of the investigations. In particular, cases involving consideration of changes in temperature, yielding supports, errors in workmanship, and initial stresses generally, are dealt with clearly and readily. Mr. Cilley.

Turning from general considerations to the special problem offered by the design of the Kinzua Viaduct, consider the plane bent of five stories. By cutting through each of the five headers, say at the middle, separating the two halves and putting at each cut section the forces which had actually acted on it before the cutting, each of the halves would become statically determinate, and would offer no further difficulties. But these forces acting on the cut sections are precisely those which are not known, but have to be determined. At each section there are three of these, a shear, an axial stress and a moment, or fifteen in all. Indicating each of these by a letter, there would be no difficulty in writing out the expression for the potential energy of the whole bent in terms of the applied forces and these fifteen unknowns. Partially differentiating with respect to each of these unknowns, and placing each result equal to zero, fifteen linear equations between the fifteen unknowns are obtained, the solution of which yields their values.

But in Mr. Grimm's treatment not fifteen but only five unknowns appear, and only five equations are found. This is in consequence of

* The potential energy of a structure under load may be expressed by $\frac{1}{2} \sum S \Delta$ — one-half the sum of all the products of the stresses into the corresponding strains—if the material obeys Hooke's law. The strains Δ are proportional to the stresses S, $-\Delta = KS$ say—so that the potential energy of the structure becomes $\frac{1}{2} \sum KS^2$. Now, if the structure be statically indeterminate, the S are linear functions of the superabundant stresses X. The values of these, if there are to be no initial stresses, it has been seen, must satisfy the equations obtained by partially differentiating the energy equation, $\frac{1}{2} \sum KS^2$, with respect to each of them, bearing in mind that they are independent variables, and equating each result to zero. That is, the X are determined by solution of the simultaneous linear equations: $\frac{\delta (\frac{1}{2} \sum KS^2)}{\delta X} = \sum KS \frac{\delta S}{\delta X} = 0$.

But the expressions $\frac{\delta S}{\delta X}$ are the stresses S^1 which would result, if the value unity were given to the X in question, all other X being zero, and the equations become: $\sum KSS^1 = 0 = \sum \Delta S^1$; a form familiar to all who use the method of virtual work, which at once expresses the fact that the difference in displacement in the direction of X of the two sides of the section on which the stress X works, is zero.

Mr. Cilley. his neglect of certain factors. If the complete equations were written out, denoting by Y and Z the axial stress and central moment, respectively, of each header, it would be found that, owing to the symmetry of the structure and the character of the loading, the equations $\frac{\delta F}{\delta X} = 0$ do not contain either the Y or the Z, while the equations $\frac{\delta F}{\delta Y} = 0$ and $\frac{\delta F}{\delta Z} = 0$ do not contain the X; F representing the expression for the potential energy of the bent under the given loading. Mr. Grimm considers only the first set of equations. The second and third sets determine values of Y and Z largely dependent on the compressibility of the headers. Assuming the headers incompressible (although flexible) the Y and Z become zero, and this, at least, is involved in Mr. Grimm's solution. He further treats the columns as incompressible and inextensible, although flexible. Mr. Grimm's statements in this connection on page 29 are very misleading.

Turning to Mr. Grimm's actual work, it may be noted that he has integrated for the vertical distance between headers, instead of for the true oblique lengths of the columns. These and other errors in his work, made apparent by a calculation of the deflections from his results, which wholly failed to check, made it futile to attempt any thorough discussion of Mr. Grimm's design from his own data, and the computation of correct data is too laborious.

On two points, however, it is important to arrive at some sort of a conclusion: As to the economic necessity of the design used, and as to its stiffness.

As to the first of these, the writer finds that by Mr. Grimm's data the unit stress in the headers rises to some 12 000 lbs. Taking this as a basis, and the column formula as published, the writer computed a stiff system of diagonal bracing at about 45° with the vertical, and found it at least as light as the headers. So that, even without considering the removal of the secondary stresses in the columns (some 5 000 lbs. per square inch) resulting from the use of the stiff diagonal system, it is at least as economical as that actually used, thus showing that Mr. Grimm was mistaken in supposing his design one required for economy.

As to the second, it apparently does not require argument to demonstrate that a completely braced frame must have smaller deflections, for the same unit stresses, than a frame without diagonals, resisting only through the bending of relatively slender members. Mr. Grimm, at least, can hardly take exception to this conclusion in this case, for in his work he has gone on the basis that the distortions due to bending are so great that those due to axial stress are by comparison wholly negligible. Either this is correct or Mr. Grimm's computations are worthless.

C. R. GRIMM, M. Am. Soc. C. E. (by letter).—In presenting a paper on the Kinzua Viaduct the writer had hoped that its discussion would produce some points of value, either of a practical or theoretical nature, but in this he has been disappointed. Mr. Grimm.

If the design had been subjected to a critical and scientific examination, facts would have come to light instead of opinions.

While an answer to the minor objections of the critics can be found in the paper, and need not be touched upon again, the writer will confine himself to answering more fully the objections on which the discussors lay the greatest stress, as this is, in any case, the object to be attained.

Answers to objections of the same nature, although raised by several discussors, will, of course, be given but once.

Some mathematical notes in regard to the analysis have been added in order to meet the wishes of some of the discussors and give a proof of certain statements made by the writer.

Although Mr. Buck failed to do justice to the practical part of the design, the writer gives him credit for his persistent effort, with able outside assistance, to follow the analysis, but cannot agree with him that sufficient data for a discussion have not been given.

As previously stated, the writer assumed the reader's familiarity with the principle of least work, and engineers acquainted with modern analytical methods will find the data sufficient as a basis for a thorough discussion. It has been stated in the paper that wind stresses alone would be examined, and, of course, by some analytical method. The assumption of the points of contraflexure in the graphical method is arbitrary, while in the analytical method this assumption is not and must not be arbitrary. As a matter of fact, no assumption whatever has been made in the analysis that cannot be proved to be justifiable; but this point will be taken up at the proper place.

In reference to the closing paragraph of the paper, it should be stated that by a group of members is meant either the columns or the cross-struts, and, although the effect on the resulting moments of a change in the moments of inertia of a group of members between rather wide limits is quite small, the effect on the stiffness of the frame may or may not be small. This depends upon how the increase in the moments of inertia is effected; if, for instance, by increasing the depths of the struts or of the columns, then, of course, the bent will gain in stiffness.

According to the figures on the stress sheet, there is no up-lift on the windward column, as erroneously stated by Mr. Buck; but the writer agrees with him in the rejection of the old anchor bolts. Strong bolts should have been used, with superior details, and, therefore, much greater efficiency.

Mr. Grimm. Mr. La Chicotte laments over both the length of the analysis, as the principal feature of the paper, and the shortness of the description of the viaduct, but, to judge from his vigorous complaints that so much has been left unexplained in the analysis, one is led to the belief that the length and breadth of the analytical method is far behind the requirements of Mr. La Chicotte, and it is significant indeed that his whole discussion centers in the bottom transverse strut. Mr. La Chicotte, in conjunction with Mr. Moisseiff and Mr. Buck, objects to the writer's assumption of the fixity of the bents.

While perfectly aware of the fact that no connections whatever are fixed, in the scientific sense of the word, the writer, nevertheless, is unshaken in his view that, at least in the case under consideration, this assumption answered all practical purposes.

If the discussion was meant to be logical, why, then, was not the writer questioned with regard to the different degrees of fixity of the bents? Why was he not questioned about the influence on the moments of the varying heights of the bottom stories of the towers; and, consequently, why was he not expected to undertake forty sets of computations instead of the five actually made? And why was no inquiry made regarding the assumption of concentrated wind pressures? Drawing the final conclusions, which Mr. La Chicotte from some reason ignores, we arrive at the following results.

Assuming his figures to be correct, then the bending stress in the bottom column of the highest bent, according to his own data—and no other members need be considered—is increased 1 600 lbs. per square inch, which means that the computed wind stress, by Mr. La Chicotte, could be increased 360% before the combined stress from the different loadings would reach the elastic limit, not considering the reducing effect of an 8-ft. strut on the moments of the column.

While an inquiry into the assumption of fixity is quite proper, it is nevertheless true that Mr. La Chicotte has made a mountain out of a molehill, and the work spent on the bottom strut should have been directed toward some more profitable point.

Mr. La Chicotte claims that he has worked out the problem for himself. The only thing he has done consisted in trifling additions (on account of the transverse bottom struts) to the writer's original condition equations, and solving them over again. The insufficient and partly erroneous general theoretical remarks, together with his statements about the points of contraflexure, which are bound to lead to erroneous views, fully explain that he did not digest the problem.

A solution of the problem on independent lines would have been very welcome to the writer for the sake of comparing results, but no such solution has been forthcoming during this discussion.

The writer agrees with Mr. Lindenthal when he questions the advisability of building very high viaducts. A viaduct of 300 ft. in

height and with spans of some 40 and 60 ft. has already passed the limits of economy and desirability, no matter how it is designed, and, in the opinion of the writer, in the present case from a purely economical point of view, its place should have been taken by an arch. A spandrel-shaped arch, with its horizontal top chord, could hardly be considered seriously, as it is neither fit for a very long span nor for a high rise. Of all the different types of arches, the writer, in all probability, would have selected as the most suitable the crescent form, the chords of which intersect at the springing. This form is the most economical among all the two-hinged statically indeterminate arches, and is excellently adapted for a long span and a high rise. With moderate temperature stresses it combines great uniformity in the chord stresses, so that each chord could be built of uniform section from end to end and without any great waste of material. The approaches could be viaducts, the erection of the arch be effected cantilever-fashion, and the trusses be given an inclined position, whereby any amount of lateral stability is obtained. Mr. Grimm.

Mr. Lindenthal's idea, to have the Kinzua Viaduct designed to last for centuries, cannot be taken seriously, for very obvious reasons.

The doubling of the traction stresses, as Mr. Lindenthal thinks, is not warranted by the facts. To a given traction stress corresponds a longitudinal deflection of the top of the tower, and, while the play of the intermediate spans is sufficient for the changes of temperature, it is entirely inadequate to double the traction stresses.

One of his statements in connection with this subject reads: "There can be no bending moment at the top of the towers worth considering;" and, a few lines below, there is the following question:

"And does not the bending of the towers subject the riveted connections (of the track girders) at their tops to large bending strains, for which these connections are not adapted?"

The writer believes that Mr. Lindenthal will find it hard, if not impossible, to give a satisfactory explanation for this bare contradiction.

A reversal of stresses in some parts of the towers occurs only under extreme conditions and at rare intervals, so that Mr. Lindenthal's apprehensions are more imaginative than real. The change in length of the track stringers, due to a change of temperature of 75° Fahr., is about 10 ins., and, since the structure is only about 2 000 ft. long, Mr. Lindenthal thinks that one dilatation joint at each end is sufficient to take care of the longitudinal motion, but as he fails to explain in what manner these expansion joints are supposed to annul the effect of a longitudinal force of about 800 tons on the viaduct, the writer cannot discuss his suggestion. His cloudy views of, and prejudice against, the general design, have prompted him to a criticism which is not justified by the facts involved in this case.

Mr. Grimm. With regard to the omission of diagonals in the transverse faces of the towers, he asks:

"Does that arrangement produce a more rigid construction laterally? It is obvious that a deflection diagram of the tower bents at right angles to the track would furnish the emphatic answer 'No.'"

A mere dictum is not equivalent to a proof. No engineer will earnestly dispute the fact that he only is entitled to an extreme judgment who combines his practical experience with the most searching analytical examination of a structure, by which means its true nature is disclosed. Mr. Lindenthal has not even given the least demonstration of any sort, not to mention a rigid mathematical discussion, to prove the truth of his assertion.

In view of these facts the writer rejects the above quoted statement as also other bold but groundless statements, as worthless.

Mr. Cilley complains about the nature of the analysis used. To characterize his standpoint it will be best to quote his own statements from different sources. In the discussion of his paper on Indeterminate Frameworks,* he expresses himself as follows:

"The use of methods of calculation based on the principle of virtual 'Velocities' or 'Displacements' or 'Work,' as it is variously termed, or which result from the principle of least work, which is simply an integral form of the preceding, is the simplest as well as the most correct means of determining changes in stress and strain in indeterminate structures. Where such structures exist, and, therefore, must be examined, the writer most strongly approves the use of these methods. And in designing, if engineers will use indeterminate forms, these methods are still to be preferred."

Apparently, his mind has undergone a remarkable change in reference to the principle of least work, for he writes in the present discussion regarding it:

"What are its merits and demerits as a working method? it may be asked. It may be used almost mechanically, and therefore saves thought, but at the expense of understanding. Like most integral theorems, it is a sort of blanket which hides the details and prevents the perception of the significance and purpose of the procedure."

In respect to the diffusion of the principle of least work, Mr. Cilley is quoted from the first-named source, when he says of himself: "The writer's point is that while the methods just referred to are the best known," etc., but in the present discussion he speaks of the principle of least work as of "a principle so little known and less understood," etc.

It is to be regretted that he did not put on record some illustrations of "almost mechanical" applications of the principle of least work in the solution of engineering problems. Comments on his contradictions are superfluous.

* *Transactions*, Am. Soc. C. E., Vol. xliii, page 426.

Mr. Grimm.

Mr. Cilley accuses the writer of having committed an error on account of the integrations for the vertical heights instead of for the lengths of the columns. The writer did integrate for the vertical heights for the sake of simplification, which involved no error at all in that sense underlying Mr. Cilley's remark. Let us assume a beam, loaded at its extreme end with a horizontal force P (see Fig. 15). The determination of the horizontal deflection δ in the direction of P requires an integration for the axis of the beam, but, if the angle α is not too large, we integrate for the vertical height for the sake of simplification, and while it is true that an error has been committed, for all practical purposes, this error can be neglected as being too small to be considered. In the examination of flat arches, for instance, we proceed in like manner and use the differential dx instead of the arc element ds. Similar reasons prompted the writer to perform the integrations as he did. The same misleading statement is repeated by Mr. Cilley in regard to the unknowns of the bents, when he says of the writer: "This is in consequence of his neglect of certain factors."

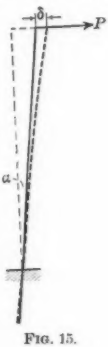

FIG. 15.

Mr. Cilley seems to be unconscious of the fact that the influence of axial and shearing forces on the final result is small compared with that of the bending forces, a fact so widely known that it is wasting time to dwell on this subject longer.

We will now turn our attention to the omission of diagonals in the transverse faces of the bents and to the deflections of the towers, treating the latter as free standing cantilevers, although, strictly speaking, they are not.

Believing that a comparative computation of the deflections would be welcome, the writer examined the deflections due to bending of five different bents, the same as are described in the paper, and also the deflection of an articulated bent, corresponding in its height, 155 ft. 2 ins., to a three-story bent as built, but having five stories instead of three, for obvious reasons (see Fig. 17).

The wind load is, of course, the same for all bents, and equal to that assumed in the paper.

Each half of a bent is statically determinate; and, the exterior forces being all known, the deflection can be determined.

The calculations of the deflections proceed from the "Theorem of the Derivative," which is expressed as follows: The displacement, in the direction of an external force, of its point of application, equals the partial derivative with respect to the force of the work of deformation of the frame.

Mathematically expressed, the deflection D_1 of the stiff bent sub-

Mr. Grimm.

jected to moments and D_2 of the articulated bent appear under the form

$$D_1 = \int \frac{M}{EI} + \frac{\delta M}{\delta P} dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and

$$D_2 = \Sigma \frac{S L}{E A} + \frac{\delta S}{\delta P} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In Equation (2), S designates the stress in any bar; L the length of that bar; and A its sectional area. The other letters in Equations (1) and (2) have the same meaning as in the paper.

The stress S can be expressed:

$$S = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \ldots . \alpha_m P_m + \ldots . + \alpha_n P_n;$$

and the partial differential coefficient, with respect to any of the external forces, as for instance P_m, would be:

$$\frac{\delta S}{\delta P_m} = \alpha_m,$$

α_m is the stress in any bar of the frame, if all the exterior forces P are supposed to be removed, excepting P_m, for which is substituted a force $= 1$.

The meaning of α_1, α_2, α_3, etc., is similar to that of α_m.

The term $\frac{S L}{E A}$ gives the alteration in the length of any bar in the frame, and if we substitute for this term the play in the pinholes, Equation (2) can be used to determine both the elastic and the non-elastic deflection. In Fig. 17 the stresses "α" are put in brackets.

The integrations or summations, as the case may be, must be extended over the whole frame.

Since the lengths are given in feet and the forces in tons for the stiff bent, the moments of inertia must be reduced to feet and the modulus of elasticity to tons and the square foot.

The deflections of the tops of the five bents, computed from the data given in the paper, are as follows:

1-story	bent,	31 ft.	2 ins.	high	on	centers,	$\frac{1}{16}$ in.
2	"	93 "	2 "	"	"		$\frac{13}{32}$ "
3	"	155 "	2 "	"	"		$\frac{9}{16}$ "
4	"	217 "	2 "	"	"		$1\frac{1}{32}$ ins.
5	"	279 "	2 "	"	"		$2\frac{1}{4}$ "

The total deflection, elastic and non-elastic, of the articulated bent amounts to $1\frac{15}{32}$ ins.; the play in the pinholes is supposed to be $\frac{1}{32}$ in., while that of a three-story riveted bent of triangular form is only $\frac{9}{16}$ in.

The assumed sections of the columns in the articulated bent are the same as those in the riveted one, but, if such a frame had been built, no such sections would have been used; on the contrary, the

Mr. Grimm.

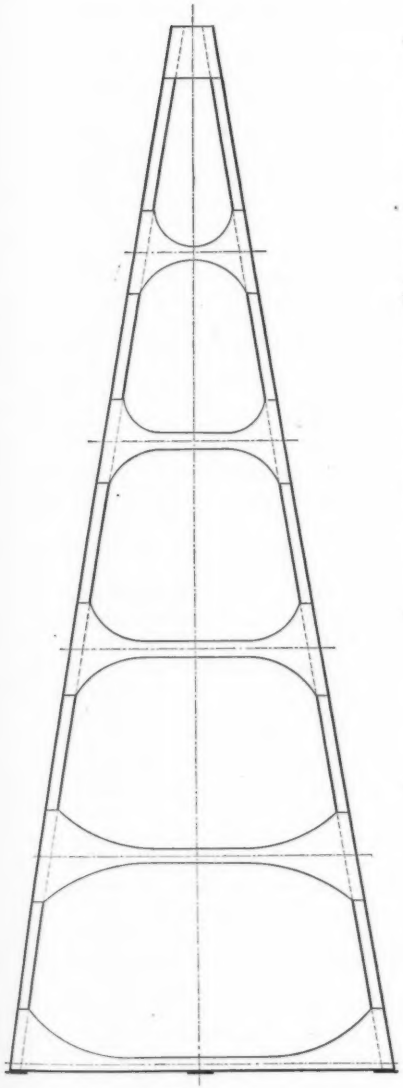

FIG. 16.

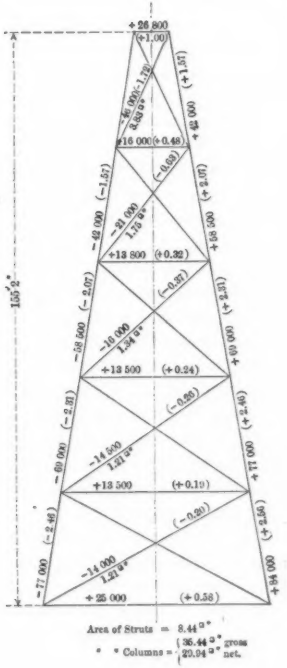

FIG. 17.

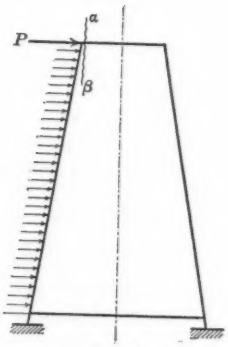

FIG. 18.

Mr. Grimm. column sections would have been much smaller, corresponding to a larger deflection than that of $1\frac{15}{32}$ ins.

The deflections compare very favorably indeed with those of any other form of bent, articulated or not, and this circumstance is due to its triangular form and deep and very rigid header.

Every frame has somewhere a critical point, so that if a force is applied at this point, it will show its least resistance to distortion.

For a bent of the usual form, this critical point is at the top, and, unfortunately for this bent, it is at the top where the principal wind forces, as well as all the dynamic forces, are applied.

This statement is not applicable in its full force to the present bent, because a riveted triangular frame shows its greatest resistance to distortion if forces are applied at the apex.

While it is apparently evident in itself for the critics in general, requiring no demonstration of any sort, that a triangular bent deflects very considerably more than an isosceles trapezoid, the mathematical examination shows this view to be erroneous. Articulated high bents are capable of large swaying motions, as it were, but a stiff and massive triangle, the simplest form of a bent, with a very rigid apex, the principal point of application of the forces, properly designed, with due regard to minimizing the bending stresses and the deflections, will perform legitimate vibrations.

The wind stresses in the transverse diagonals are comparatively small, and decidedly so toward the bottom of the towers. The questions may now be asked: Is it really such an undertaking, as some of the critics have imagined it to be, to replace these diagonals, with their small stresses, by massive fixed connections, at the same time changing the bent, a very important and very advantageous step, into a triangle? And is a frame of a few, but very substantial, members not more fit to stand the effect from the passage of trains, than a frame of very many but rather light members?

If the present bents would leave something to wish for, that, in the writer's opinion, would be no argument against their use, because every design is capable of improvement.

Although the workings of a frame can only be judged by the analysis, Mr. Morison nevertheless clearly recognized the importance of the principal features of this design, when he made the statement that it is possible to build such a structure perfectly secure; and, coming, as it does, from so distinguished an engineer, it commands respect.

It is a well-known fact that the requirements of economy call for the least number of substantial members in a structure, instead of for a multitude of slender pieces; and, in order to make a fair comparison, bents as well as towers should be compared with respect to weight, ease and quickness of manufacture and erection, etc.

It does not seem to need an argument to show that the sub-

stitution of two compression diagonals for an intermediate strut can only be made at an expense of metal—and in point of weight it would be still worse if two diagonals together with a strut were used—as the average combined length of the two diagonals in each story is about three and one-half times as great as the length of the strut. The weight of the web members in the transverse faces of high bents of ordinary construction is entirely out of proportion to the stresses in these members, and the weight of that mass of longitudinal secondary bracing used as a prop to prevent the transverse members from a collapse, is a very large item indeed in tall towers. In such a case, where is the economy? It must also be borne in mind that in the present case a considerable weight could have been saved if the towers had not been built all alike in their sections from their tops down. Mr. Grimm.

While stresses and sections in a bent, as usually built, are entirely out of proportion, there is in the triangular form, on the whole, a far better harmony. The intermediate struts in the Kinzua Viaduct have been built with parallel chords, for practical reasons, giving them a surplus of metal around their centers, where it was not necessary. These struts do not need to be straight. Fig. 16 shows the highest bent with curved cross-pieces, supposed to be box-shaped, struts and brackets all in one piece and fork-like at the ends, so that they can take the firmest grip on the columns.

In a design like this the metal in the struts can be distributed so as to meet in the best possible manner the requirements of the stresses. It is also quite feasible to design a true triangle by making the longitudinal axes of the columns to intersect on the horizontal top edge of the header, the latter to be cantilevered to take the stringers.

The question whether a triangle can be so designed as to carry a double-track railway, is best decided by an investigation of the stresses and details.

An increase in the stability by using a greater batter involves a considerable outlay of money for the usual form of bent, but not so for a portal-braced triangle.

If intermediate struts with curved ends were used, as shown in Fig. 16, the increase in weight of these struts would be effected by lengthening the central parts, where the struts are the shallowest, and, although the length of the columns is somewhat increased, it must not be forgotten that the bending moments of the frame decrease—as also the deflections—with an increase of batter; so that, on the whole, a greater stability can be obtained with the portal-braced triangle at a lesser cost.

There are two more points not mentioned by the discussors.

Although we hear a good deal about the statical determinateness of towers, as ordinarily built, they are in reality statically indeterminate, and even, quite often, highly so.

Mr. Grimm. Omitting the intermediate struts in either the longitudinal or transverse faces, and using in each story two crossed diagonals and a strut at top and bottom, we have a singly statically indeterminate frame, and if, besides the diagonals, intermediate struts are used, as is done very often, we produce a frame that is n fold statically indeterminate, if there are n stories.

The second factor to be dealt with in such a frame is the bending stresses due to the fixed joints. As a matter of fact, the constructor completely ignores the statical indeterminateness in his computations by the simple supposition that it is non-existent, assuming frictionless pins instead of fixed joints.

Although it is true that some constructors minimize the bending stresses by skilful designing, others, apparently, do not pay the slightest attention to them.

In a case like that under discussion, all stresses must be examined, and herein the writer sees an advantage.

The second point deserving to be mentioned is the appearance of a portal-braced triangle, which is far ahead of that of a very slim and ugly looking tower with its many pieces, impressing one more with the idea of a huge bird cage than anything else. In concluding this discussion it will be necessary to add some notes with respect to frames, the diagonals of which have been replaced with fixed joints.

The frames considered are supposed to be symmetrical in form as well as in distribution of material about a vertical axis; and the forces applied to them are assumed to act in the planes of these frames.

In the paper the writer stated that a one-story bent, fixed at the bottom, is a threefold statically indeterminate structure. Assuming for the present that the exact values of the three unknowns in the top story of a two-story bent were known and these unknowns as exterior causes applied to the bottom story, there would still be three more unknowns left to be determined, so that we have six unknowns in all for a two-story bent. And, in general, a bent of n stories has $3n$ unknowns. The causes which may influence a truss of this kind are as follows:

1.—Bending forces.
2.—Axial forces.
3.—Shearing forces.
4.—Uniform change of temperature.
5.—Non-uniform change of temperature.
6.—Displacements of the points of application of the reactions.
7.—Misfits of the bars composing the frame.

The condition equation, covering the foregoing seven causes, is:

$$R = \int \frac{M}{IE} \frac{\delta M}{\delta X} dx + \int \frac{F}{EA} \frac{\delta F}{\delta X} dx + C \int \frac{T}{E_t A} \frac{\delta T}{\delta X} dx +$$
$$\int \varepsilon \, t_o \frac{\delta F}{\delta X} dx + \int \varepsilon \frac{t_1 - t_2}{h} \frac{\delta M}{\delta X} dx.$$

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In this equation:

M = a bending moment;

F = a direct stress or axial force;

T = a shearing force;

I = a moment of inertia;

A = an area;

ε = the coefficient for extension or contraction due to a change in temperature of 1°;

t_0, t_1 and t_2 = temperatures according to certain specified assumptions and laws;

C = a constant, dependent on the form of a section;

E = the modulus of elasticity, expressing relations between longitudinal strains and normal stresses in the same direction;

E_t = the transverse coefficient of elasticity, expressing relations between distortions and tangential stresses in the same direction.

For steel, $$E_t = \frac{3}{8} E \text{ to } \frac{2}{5} E.$$

R is the work of the reactions done under the supposition that any one of the unknowns, $X = 1$.

The first integral refers to bending moments, the second to axial forces, the third to shearing forces, the fourth to a uniform change in temperature, as also to misfits, the fifth to a non-uniform change of temperature.

It is customary in a case like that under consideration to reject any possible influences due to the causes under 4, 5, 6 and 7. Of the remaining causes, under 1, 2 and 3, the bending forces are by far the most important, as the influence of axial and shearing forces on the final results is too small to be considered. A few examples may suffice to illustrate the nature of the bents.

The two-story frame, Fig. 20, is acted upon by two concentrated loads, one at each panel point, 4 and 2, respectively, and, in order to avoid extensive calculations, simple dimensions have been selected, with the assumption of a uniform moment of inertia. We will consider bending alone.

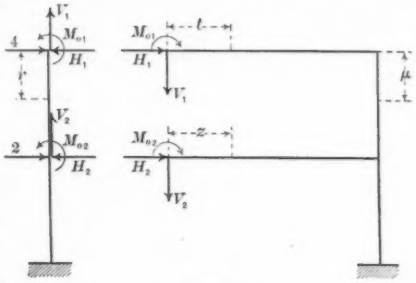

Fig. 19.

Passing a cut very close to the left column and applying at each cut and at each of the two fragments, but in opposite directions, a

Mr. Grimm. moment, a shearing force and an axial force (see Fig. 19), the equilibrium is re-established and the six condition equations can be written down at once and solved.

These six equations are:

$$\left.\begin{aligned} &\int \frac{M}{EI}\frac{\delta M}{\delta M_{o1}}dx=0;\int \frac{M}{EI}\frac{\delta M}{\delta M_{o2}}dx=0;\int \frac{M}{EI}\frac{\delta M}{\delta V_1}dx=0;\\ &\int \frac{M}{EI}\frac{\delta M}{\delta V_2}dx=0;\int \frac{M}{EI}\frac{\delta M}{\delta H_1}dx=0;\int \frac{M}{EI}\frac{\delta M}{\delta H_2}dx=0. \end{aligned}\right\}$$

Each of these six integrals must be applied to each bar of the frame, the results added together and put equal to zero, so that, in all, six new equations are obtained, from which the unknowns can be computed. To illustrate the foregoing statement, take as an example the partial differential coefficient of the work of deformation of the frame with respect to V_1. The force V_1 leaves Bar 2 entirely unaffected, produces a direct stress in Bars 3 and 4 and bending in Bars 1, 5 and 6.

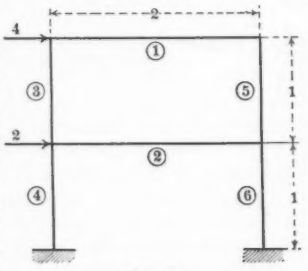

FIG. 20.

With this understanding, we have:

$$\int \frac{M}{EI}\frac{\delta M}{\delta V_1}dx=\int_0^2 \frac{M_1}{EI_1}\frac{\delta M_1}{\delta V_1}dt+\int_0^1 \frac{M_5}{EI_5}\frac{\delta M_5}{\delta V_1}du+$$

$$\int_1^2 \frac{M_6}{EI_6}\frac{\delta M_6}{\delta V_1}du=0.$$

The indices refer to the bars, and, under the above assumption,

$$I_1=I_5=I_6.$$

The resulting six equations, after performing the computation of the integrals, are:

$$\left.\begin{aligned} -18\,M_{0_1}-6\,M_{0_2}+32\,V_1+12\,V_2-12\,H_1-3\,H_2&=0\\ -6\,M_{0_1}-12\,M_{0_2}+12\,V_1+20\,V_2-9\,H_1-3\,H_2&=0\\ +12\,M_{0_1}+9\,M_{0_2}-12\,V_1-9\,V_2+16\,H_1+5\,H_2-37&=0\\ +3\,M_{0_1}+3\,M_{0_2}-3\,V_1-3\,V_2+5\,H_1+2\,H_2-12&=0\\ +6\,M_{0_1}+2\,M_{0_2}-6\,V_1-2\,V_2+4\,H_1+H_2-9&=0\\ +2\,M_{0_1}+4\,M_{0_2}-2\,V_1-4\,V_2+3\,H_1+H_1-7&=0 \end{aligned}\right\}$$

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Solving for the unknowns we have:

$$V_1 = \frac{270}{228};\ H_1 = 2;\ M_{0_1} = \frac{270}{228};$$

$$V_2 = \frac{396}{228};\ H_2 = 1;\ M_{0_2} = \frac{396}{228}.$$

Fig. 21 shows one of the intermediate panel points in equilibrium under the influence of the exterior and interior forces. The small circles indicate the points of contraflexure, which, for the struts, it will be noticed, are at their middle. The compression in the struts equals one-half of the exterior concentrated loads.

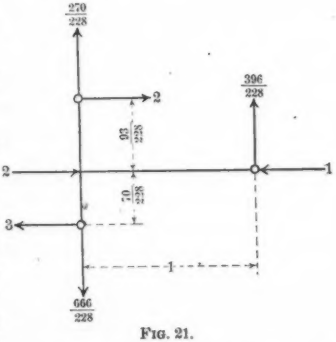

FIG. 21.

A variation in the moments of inertia between columns and struts, although of course influencing the bending moments, has no influence on the location of the points of contraflexure of the struts. But, if besides the bending forces, the axial forces had also been considered, the points of contraflexure of the struts would have been thrown somewhat out of their middle.

If the influence of bending forces alone is examined, a procedure perfectly legitimate and answering all practical purposes, and if it is known in advance that under this condition the points of contraflexure of the struts are at their middle and their compression equal to one-half of the panel point concentrations, a very considerable labor indeed can be saved by computing one unknown for each story instead of three, as the writer actually did. Passing a vertical cut through the center of the frame, Fig. 20, and applying V_1 and 2, as also V_2 and 1 to each half of the bent, but in opposite directions, disregarding the moments at the cut, as no moment can pass it, we arrive very quickly at the two equations:

$$\left.\begin{aligned} 14\ V_1 + 6\ V_2 - 27 &= 0 \\ 6\ V_1 + 8\ V_2 - 21 &= 0 \end{aligned}\right\}$$

Solving for V_1 and V_2, we find, as before:

$$V_1 = \frac{270}{228} \text{ and } V_2 = \frac{396}{228}.$$

Fig. 18 shows a one-story bent of the viaduct, loaded with a concentrated wind pressure of 13.4 tons at the top and a uniform wind pressure of 30 lbs. per square foot on the column.

The influence of the bending forces alone will be examined.

Separating the bent by a cut "$\alpha - \beta$" at the end of the header,

Mr. Grimm. and applying at the cut and at each fragment, but in opposite directions, a moment M, a transverse force V and an axial force H, we obtain the following three equations:

$$\left.\begin{array}{rrrrrrr} -603 & V+ & 126\,M+ & 1942 & H= & 13\,515 \\ +9922.6 & V- & 603\,M- & 9253 & H= & 47\,010 \\ -9253 & V+1\,942 & M+40\,339.5 & H= & 282\,059 \end{array}\right\}.$$

The values of the unknowns are:

$V = 15.8$ tons; $H = 7.07$ tons; $M = 73.7$ foot-tons.

The point of contraflexure, in this particular case, is moved a little distance out of the middle of the header, and this fact is due alone to the influence of a uniformly distributed wind pressure.

If the bent is influenced solely by the concentrated wind pressure of 13.4 tons, the three equations will be:

$$\left.\begin{array}{rrrrrrr} -603 & V+ & 126.5\,M+ & 1942 & H= & 13\,011 \\ +9922.6 & V- & 603 & M- & 9253 & H= & 45\,046 \\ -9253 & V+1\,942 & M+40\,339.5 & H= & 270\,275 \end{array}\right\}.$$

and the unknowns are

$V = 15.2$ tons; $H = 6.7$ tons; $M = 72.4$ foot-tons.

These are the same results as given in the paper. The point of contraflexure of the header is at its middle, and the compression of the header equals one-half of the wind load. For all practical purposes, the concentrated wind pressure will be found to be sufficiently exact.

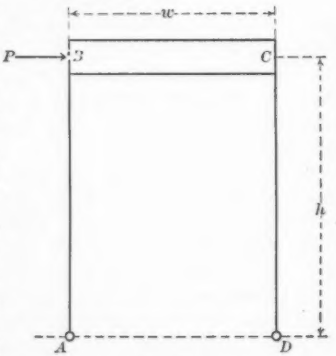

FIG. 22.

The very small influence of axial forces, with the consequent shifting of the points of contraflexure of the cross-pieces out of their middle is shown by the next example (see Fig. 22). $A\ B\ C\ D$ represents a hinged bent with a cross-strut or a floor beam rigidly riveted between the tops of the columns, and acted upon by a single concentrated force P at the top. This bent is singly statically indeterminate.

Let w = the width of the bent;
h = " height " " "
I = " moment of inertia of $A\ B$ and $C\ D$;
I_2 = " " " " " $B\ C$;
A = " area of $B\ C$;
M = any bending moment;
F = " axial force;
E = the constant modulus of elasticity;
H = the thrust at the hinge D (unknown).

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The condition equation is

$$\int \frac{M}{E\,I}\,\frac{\delta M}{\delta H}\,dx + \int \frac{F}{E\,A}\,\frac{\delta F}{\delta H}\,dx = 0.$$

The writer's investigation gave the following result:

$$H = P\,\frac{\frac{1}{2} + \frac{h\,I_1}{3\,w\,I_1}}{1 + \frac{2\,h\,I_2}{3\,w\,I_1} + \frac{I_2}{A\,h_2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The third term in the denominator is due to the influence of axial forces, and, neglecting this influence, Equation (3) becomes

$$H = P\,\frac{\frac{1}{2} + \frac{h\,I_2}{3\,w\,I_1}}{1 + \frac{2\,h\,I_2}{3\,w\,I_1}} = \frac{P}{2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Equation (3) shows:

1.—That the influence of axial forces is small on the final results, and

2.—That by their action the point of contraflexure is but slightly moved out of the middle of the cross-pieces.

Equation (4) shows that the point of contraflexure of the cross-piece is exactly at its middle, if axial forces are neglected. A two-story hinged bent has four unknowns, but if we know in advance that the consideration of bending forces alone, together with concentrated exterior loadings at the panel points, leads to the position of the points of contraflexure of the cross-pieces in their middle, we can treat such a bent as a singly statically indeterminate structure.

Of course, causes other than the axial forces, as also a change in the exterior loads, will cause the points of contraflexure to move out of the middle of the cross-pieces. Investigations like the foregoing will lead to considerations of trusses with diagonals, be they cantilevers or not.

Much more could be written, if time permitted, but it is believed that enough light has been thrown on the subject of the paper to show that it cannot be disposed of by mere opinions or assertions, without presenting argument or evidence.

In conclusion, attention is called to a comparison of the analytical results with those obtained graphically, which latter are quite useful in many cases, particularly so, if a design is governed by strong specifications.

The arbitrary assumption in the graphical method, which places the points of contraflexure of the columns in their middle, deprives the problem of its last trace of difficulty, and lowers the mathematical treatment to the level of a kindergarten method.